MODELING OF MECHANICAL PROPERTIES OF COMPOSITE MATERIALS UNDER MECHANICAL VIBRATIONS

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Abstract
This article focuses on numerical modeling of the mechanical properties of new nonlinear material structures with composite carbon fibers that would be applicable to industrial plants. These types of carbon composites will be applied especially for a construction of machines with high vibration during their operations. It was assembled composite sample numerical model with a given geometry including a bonded contact with a thin piezoelectric sensor for description and evaluation of mechanical properties. The results were compared with the measurements and statistically analyzed. It can be stated that the finite element method (FEM) can be used to describe the mechanical properties of the new type of composites.

Key words: *numerical modeling; composite materials; non-linear behavior; mechanical vibration.*

INTRODUCTION

Composite materials are increasingly used for various construction applications. The reasons are evident, because they have excellent mechanical properties, but they allow the structure to keep light weight. In the design of the final composite (fibers and matrix), due to the synergistic effect, it is possible to obtain high specific properties (a high strength, stiffness, toughness) that cannot be reached by either of the input components. The synergistic effect is characterized by a known "illogical rule 2 + 3 = 7", which describes that the sum of the properties of the individual input components (fibers + matrix) results in higher values of the specific properties of the newly prepared structure. Generally, the highest specific properties can be achieved when the fibers are loaded to acting ultimate tensile stress $\sigma_M^f\Big|_{F^f \to \max}$ that is transferred by the matrix. The matrix provides not only a transfer of stress to fibers, but also the matrix creates final geometry of the composite, protects fibers against surface wear and damage, which would lead to a loss of the stability and strength of the resulting composite. A number of authors have been dealt with researches and studies of fiber-reinforced composite structures due to their potential and specific features (*Agarwal, Broutman & Chandrashekhara, 2006; Guedes, 2010; Mehar & Panda 2016; Martinec, Mlýnek & Petrů, 2015; Petrů, Martinec, & Mlýnek 2016, Teply & Reddy, 1990, Barthelot, 1999, Gibson, 1995)*. The authors agree that composite structures are unique materials whose mechanical properties cannot be generally described in an analytical or experimental manner. Theories also differ in mathematical relationships derived for unidirectional composite structures, and a creation of the comprehensive descriptions of mechanical properties for geometrically complicated structural members such as frames, springs, and beams with multi-directional fibrous is virtually impossible. This is because their properties differ significantly with the type of fiber and matrix (i.e., physical and mechanical properties, surface treatment, chemical composition, chemical bonds, density, thermal expansion, because only a slight change results in different combinations with completely different properties. This problem is further increased if mechanical properties for the application of composite materials to machines which are heavily loaded with vibrations. The properties of the composite will be determined on the basis of the modulus of elasticity by the indirect method based on the resonance properties of the composite. A comprehensive approach addressing this issue is very complex and poorly understood. However, some authors approach similar problems by measuring vibration and numerical modeling. *Panda & Singh, 2011* solve post-buckling of nonlinear shell under uniform thermal field. They developed a mathematical model for a curved panel by the help Green-Lagrange relations using higher order shear deformation theory (HSDT). They studied parameters laminate structure, geometry, dimensions, supporting and other were studied. Also influence of mesh size is studied. Obtained results are usable for nonlinear free vibration behaviour of single/doubly curved shell panel within the post-buckled state. Authors *Marjanović & Vuksanović, 2016* deals with FEM model for the free vibration analysis of laminated composite shells. In the shell a delamination is introduced. The authors use shells with different geometries like are cylinders with different height or conical cross-ply geometry. Orthotropic linearly elastic material models is applied. The solver is introduced in MATLAB. For pre/postprocessing GiD software is used. For model basic known material data are applied. The aim of models is to investigate the influence of delamination and its size on an own frequency and also mode shape. In this paper, a numerical model of a composite sample of given geometry with a thin piezoelectric sensor was established for describing and evaluating of mechanical properties.

MATERIALS AND METHODS

Experimental measurement

Measurements were performed on a square composite board with dimensions $l = 101{,}4\,mm$ and $h = 0.5\,mm$. One side was placed in the jaws so that the resulting sample had dimensions $l = 101{,}4x90\,mm$ (Fig.1 a). On the sample the piezoelectric ceramic thin plate was bonded. The piezoelectric sensors dimensions 40 x 20 x 0.3 mm^3 were placed in the middle of square. This type of measurements gives much better response because significantly higher magnitude of vibrations can be obtained. The problem consists in suitable excitation. On Fig. 1b a response on mechanical pulse excitation is seen. More sophisticated solution was done by the help of the electromagnetic hammer. The most usable solution was created using electromagnet and perm alloy target fixed on tested composite sample.

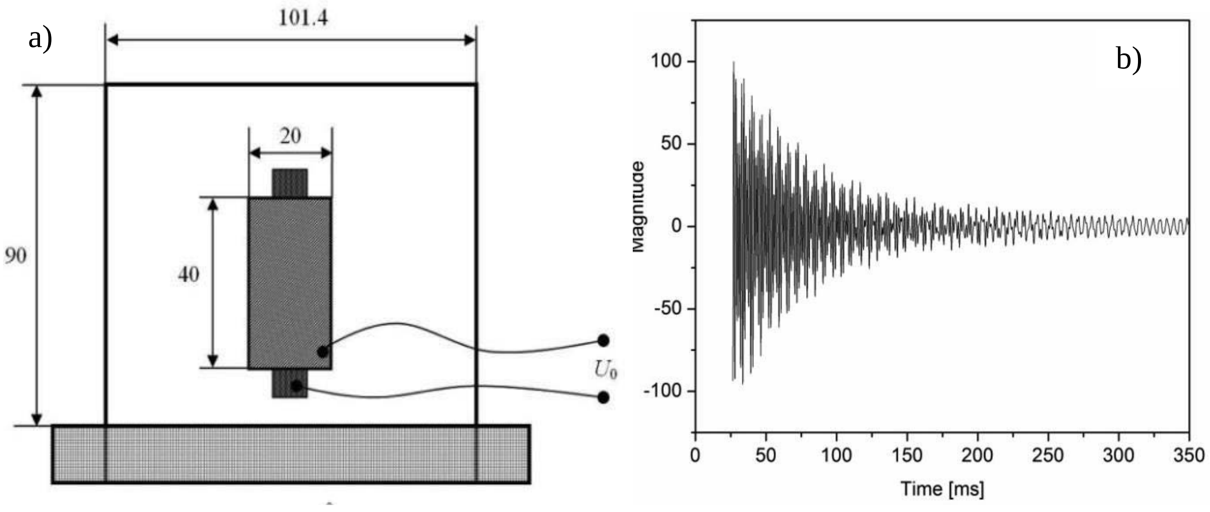

Fig. 1 Dimensions of vibrating plate from carbon composite with thin piezoelectric rectangular plate (left), An mechanical pulse excitation (right).

Numerical modelling

An accurate measurement and evaluation of the natural frequencies of composites for the purpose of a determination of mechanical characteristics such as elastic modulus, shear modulus, Poisson's ratio, and natural frequencies in composites is very difficult and complex problem. This problem consists in a suitable location and fixing of accelerometers and tensiometers. Also their weight, dimension and other properties can significantly influence measured values. Mathematical models are based on the mathematical description of the vibrating sample surface, which means you can build a matrix of elementary point sources. The displacements on the oscillating sensor surface ω in places fictitious point sources, diffuse field excited by a harmonic unit force can then be determined by equation (1).

$$w(x,y,\omega) = -\frac{F}{\rho h}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{\Theta_{i,j}(x_0,y_0)\Theta_{i,j}(x,y)}{\omega^2-\omega^2_{i,j}} \tag{1}$$

where the function $\Theta_{i,j}(x,y)$ can be for harmonic excitation expressed as

$$\Theta_{i,j}(x,y) = \frac{2}{\sqrt{L_x L_y}}\sin\left(\frac{i\pi x}{L_x}\right)\sin\left(\frac{j\pi x}{L_y}\right) \tag{2}$$

where w is magnitude of sample deflection, F is amplitude harmonic exciting force moving sample, ρ is the density of vibrating sample, h is the thickness of vibrating sample, ω is angular frequency of harmonic exciting force F moving sample, $\omega_{i,j}$ are natural modal angular frequencies of vibrating plate, x_0, y_0 are x and y coordinates of exciting force moving sample, x, y are coordinates of fictive point source on the surface of the rectangular plate, L_x, L_y are dimensions of vibrating plate, i, j are natural modes of vibrating plate in the direction x, y. For expression of the shape modes (angular resonant frequencies) of vibrating plate using (*Zizka et al., 2006*) can be established by equation (3).

$$\omega_{i,j} = \pi^2\sqrt{\frac{Eh^2}{12\rho(1-\nu)}}\left[\left(\frac{i}{L_x}\right)+\left(\frac{j}{L_y}\right)\right] \tag{4}$$

where E is modulus of flexibility (Young' modulus), ν is Poisson constant.

A development of simulation model of vibrating plate is a valuable tool for determining the distribution of stress and strain and other mechanical variables that cannot be obtained through analytical relations due to complicated shape and geometry of the real sample. The deformed shape of the surface of the sample can by analyze numerically through the finite element method (FEM - Finite element method) or the boundary element method (BEM - boundary element method). Numerical model can show founded variables as is stress, strain, deformation etc. and these values could be compared with experimentally determined values. Generally described by a system of constitutive law (5).

$$\boldsymbol{\sigma} = \mathbf{C}\cdot\boldsymbol{\varepsilon} = \begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\\ D_1\\ D_2\\ D_3\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & \cdot & \cdot & \cdot & \cdot & \cdot & e_{31}\\ C_{21} & C_{22} & C_{23} & \cdot & \cdot & \cdot & \cdot & \cdot & e_{32}\\ C_{31} & C_{32} & C_{33} & \cdot & \cdot & \cdot & \cdot & \cdot & e_{33}\\ \cdot & \cdot & \cdot & C_{44} & \cdot & \cdot & \cdot & e_{24} & \cdot\\ \cdot & \cdot & \cdot & \cdot & C_{55} & \cdot & e_{15} & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & C_{66} & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & e_{15} & \cdot & \mu_1 & \cdot & \cdot\\ \cdot & \cdot & \cdot & e_{24} & \cdot & \cdot & \cdot & \mu_2 & \cdot\\ e_{31} & e_{32} & e_{33} & \cdot & \cdot & \cdot & \cdot & \cdot & \mu_3 \end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\\ E_1\\ E_2\\ E_3\end{bmatrix} \tag{5}$$

where $\boldsymbol{\sigma}$ is stress tensor, $\mathbf{C}$ is matrix of elastic coefficients, $\boldsymbol{\varepsilon}$ is strain tensor, e_{ij} is piezoelectric matrix for stress-charge form, E_i is electric field intensity vector, D_i is vector of electric displacements, e_i is dielectric matrix with coefficients of electric permittivity on a diagonal of the matrix.

Material parameters used in FEM model for piezoceramic sensor and carbon fibers composite are introduced in Tab.1 and Tab.2. Input values were verified by FEM model (Fig.2), because material list shows different values compare to experiment. Differences were found mainly in the case of piezoelectric ceramics, which exhibits anisotropic behavior.

Tab. 1 Initial and fitted material properties of carbon fiber composite

$\rho = 1540\ kg.\mathrm{m}^{-3}$, $C_{11} = 2900\, GPa$
$C_{22} = C_{33} = 9450\ MPa$, $C_{12} = C_{13} = 5500\, MPa$
$C_{13} = 3900\ MPa$
$\nu_{XY} = 0.27$, $\nu_{XZ} = 0.27$
$\nu_{YZ} = 0.4$

Tab. 2 Initial and fitted material properties of piezoceramic sensor

$\rho = 7800\ kg.\mathrm{m}^{-3}$, $C_{11} = C_{22} = 62500\, MPa$
$C_{33} = 52\,600\ MPa$, $C_{12} = 23\,500\, MPa$
$C_{13} = C_{23} = 23\,000\ MPa \rightarrow 23\,500\ MPa$
$\nu_{XY} = 0.289 \rightarrow 0.291$, $\nu_{XZ} = 0.512 \rightarrow 0.501$
$\nu_{YZ} = 0.408$, $\varepsilon_r = 1062$, $e_{31} = e_{32} = 5.6\,\mathrm{pCm}^{-2}$, $e_{33} = -12.8\,\mathrm{pCm}^{-2}$

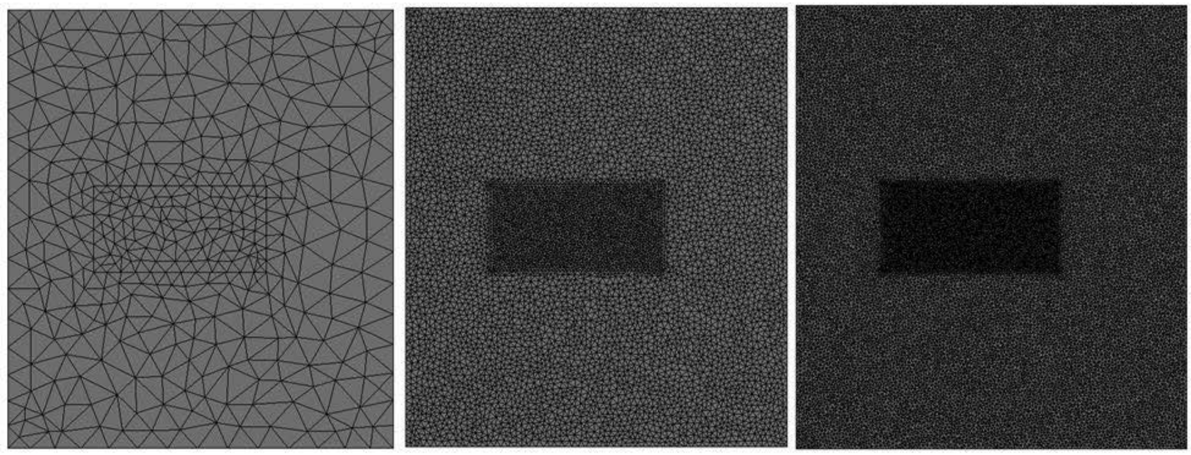

Fig. 2 FE meshing of the model in the calculation convergence: Adaptive meshing for the minimization of the residues in the solved area.

RESULTSAND DISCUSSION

For the measurement commercial impedance Analyzer HP 4192A working with 4 wire measuring system was used. The applied voltage was maximally 1.1 V. The frequency range was from 10 to 1000 Hz. The impedance method provides very low response on mechanical resonances of composite sample. This kind of measurement setup is not optimal for automatic measurement of natural frequencies of the samples. From these reasons it is not possible to use impedance measurement for evaluation of resonant frequencies by an automatic way. Therefore more suitable solution is application of an external mechanical excitation for carbon fiber composite. The problem of this type of test consists in a suitable way of the excitation. The mechanical vibration can be induced by applying a force that acts upon a negligibly short period i.e. from mathematical point of view it has character of function. It is a shock excitation, which is called the impulse excitation. The superposition principle implies that the Fourier transformation of the impulse response excitation contains wide frequency range. A FEM model of carbon fibers composite with glued piezoelectric senor plate was prepared. By FEM simulation algorithm with default values were optimized on natural frequencies 94 Hz, 240Hz and 548Hz that are in agreement with experimental measurements where piezoelectric plate is used as sensor. Both methods of measurements, direct impedance measurements using impedance analyzer, and indirect method that measures electrical response on mechanical excitation evaluated with FFT, show close values of natural frequencies. During the development and validation of methods and algorithms describing the natural frequency of composite structures is necessary to create a model that gives results close to experimental measurement and also it can describe a behavior of vibrating sample geometry of investigated composite. The model enables to compare the resulting shapes and sizes of vibration frequencies with the experimentally measured data. The model should be able to determine the spatial distribution of the displacement of the vibrating surface of the object, the stress distribution at different places and it allows the creation of graphical representation of results for comparison with the experiments (Fig.3 - Fig.5).

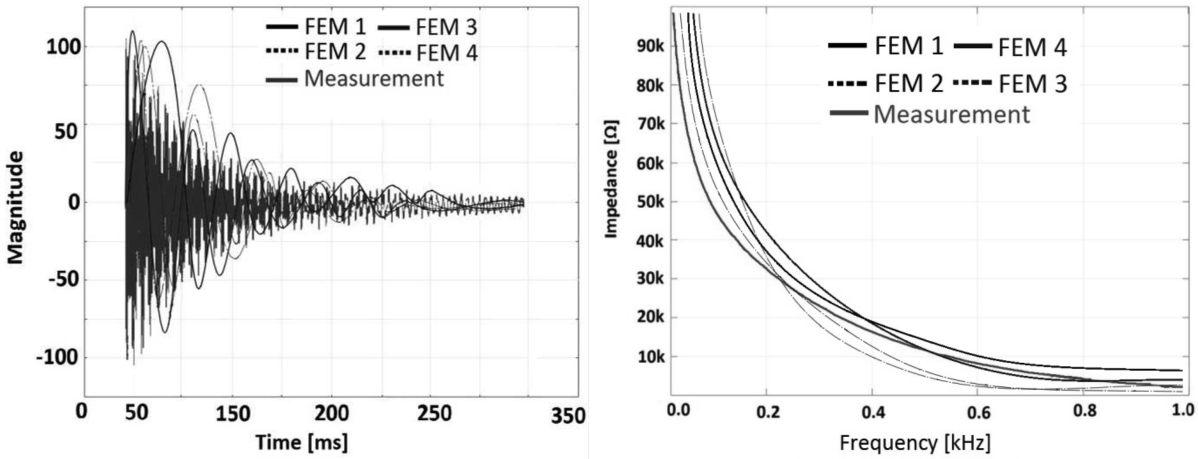

Fig. 3 Comparison of experimental input signal and FEM model for composite sample: Dependence of FEM model convergence on degree of adaptive network (left), Comparison of measurement and FEM model of impedance dependence on the frequency of alumina plate (right).

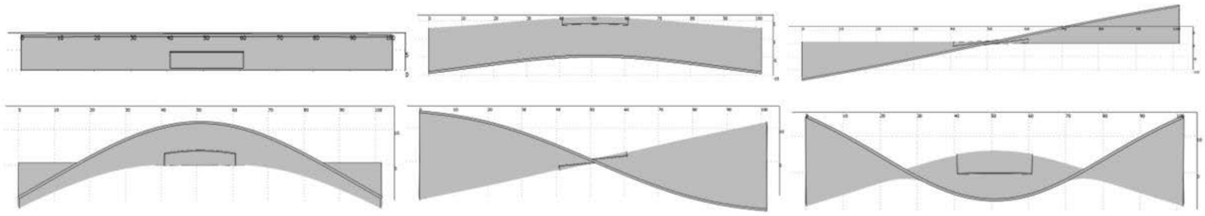

Fig.4 FEM shape mode of composite plate with piezoceramics sensor (1-6 mode).

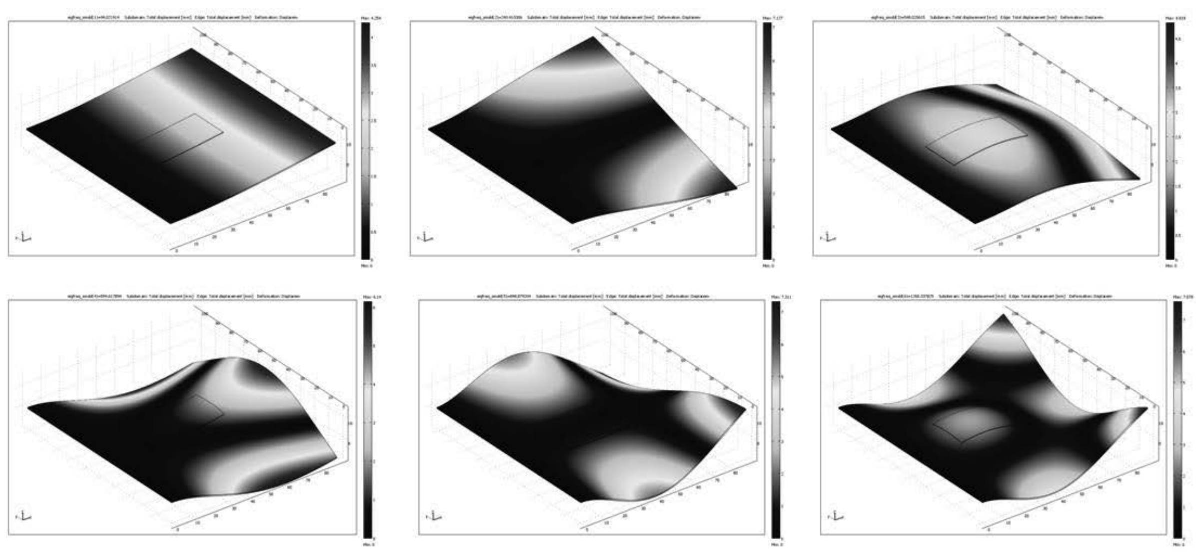

Fig.5 FEM distribution of total deformation for each mode of vibrations (1-6 mode of vibrations).

CONCLUSIONS

The main problem of measurement is small sensitivity of piezoelectric sensor at low frequencies caused probably by low energy of vibrations. Also optimal position of sensor plays an important role in sensitivity of measurements for vibration each mode. From experiments and simulations is recommended position of ceramic sensor in base (root) of vibrating strip. It is clear that electrical excitation is not strong enough especially at low frequencies. The reason is also very well visible on Fig. 5 where it is distribution of mechanical deformation on carbon fibers composite. For the second harmonic is stress very small around “vertical axes” of symmetry. It is caused by propeller like vibrating around the vertical axes. The results were compared statistically showed that the FEM model can be used for research of mechanical properties composites with carbon fibers structurally designed for industry. It turns out that these studies using FEM simulation model can contribute to the selection of an appropriate methodology for measuring mechanical properties composites are an important tool for obtaining valuable information for the design and optimization of parts construction.

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