# ANALYTICAL BEARING MODEL FOR ANALYSIS OF INNER LOAD DISTRIBUTION AND ESTIMATION OF OPERATIONAL LUBRICATION REGIME

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***Abstract***
*A mathematical model of roller bearing is presented in this paper. Calculation of load distribution and displacement is based on current standard ISO/TS 16 281. Bearing operating lubrication regime analysis is involved. It is based on lambda parameter consisting of EHD layer thickness value that is calculated from equation for wide elliptical contact and a composite surface roughness value. Model was successfully verified against commercial KISSsoft software.*

***Key words:*** *Bearing, Roller Bearing; Lubrication; EHD; Bearing mathematical model.*

## INTRODUCTION
Identification of bearing properties during operation is very important step for analysis and implementation. Since well-known classical engineering equations for calculation of bearing life does not provide a detail insight a demand for a more complex model arises. The complex model shall provide information about actual load distribution between rollers, which is based on internal geometry such as radial clearance or roller profile modification. Additionally information regarding lubrication quality based on lubricant rheological parameters, mating surface texture and operating conditions are of interest. Such model based on current theoretical findings is presented in this paper.

## MATERIALS AND METHODS
Analysis of load distribution on rolling elements within the roller bearing was carried out using method described in standard (International Organizatipon for Standardization [ISO], 2008). For roller bearings, method assumes Hertz line contact. Calculated load distribution is valid for low to medium speeds and quasi-static loading. Dynamic effects such as centrifugal and inertial forces are neglected. Another assumption concerns the deformation of outer ring/race where stiff supporting structure is considered and calculated deformations are only within the area of contact. For presented analysis, unidirectional radial load with small moment load is applied.

### Roller load distribution
Deflection of j-th rolling element was calculated from radial deflection or inner ring according to (1)

$$\delta_j = \delta_r \cos(\varphi_j) - \frac{s}{2} \tag{1}$$

Where s is a total operational radial clearance, usually obtained from catalogues and modified in order to cover its changes due to temperature deformations. Coordinate $\varphi j$ (rad) is an angular coordinate of j-th roller. For an analysis, roller was divided into 41 equally length laminas. This allows including a roller profile modification into calculation and load distribution over the roller. For current analysis, a logarithmic profile modification described as function of coordinate $x_k$ (mm) from roller center was used (2). The modification is depicted on the Fig. 5.

$$P(x_k) = 0.00035 \cdot Dwe \cdot \ln\left[ 1/(1-(\frac{2 \cdot x_k}{Lwe})^2) \right] \tag{2}$$

The deformation of each lamina including roller profile modification is calculated according to (3). If the result is negative, zero substitutes it.

$$\delta_{jk} = \langle \delta_j + x_k \cdot \tan\psi_j - 2P(x_k) \rangle \tag{3}$$

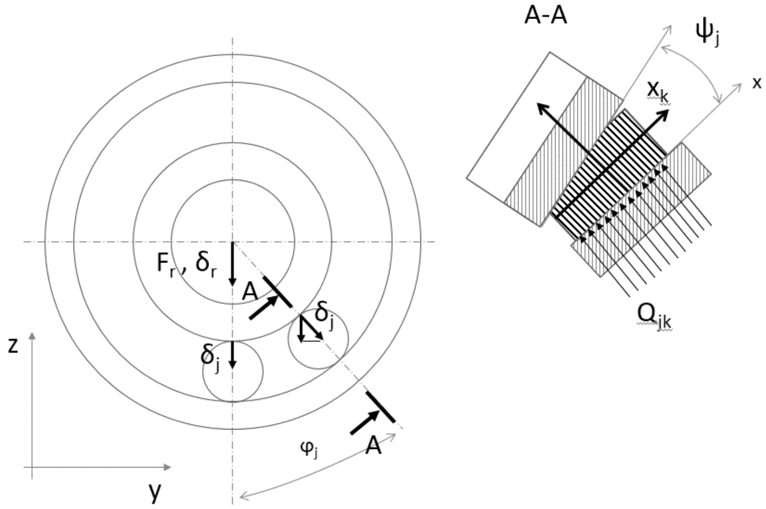


**Fig. 1** Coordinates of bearing model (ISO, 2008)

Radial load $F_r$ (N) and/or moment $M_z$ (Nm) are supported by rolling elements. The fraction of load per lamina of each rolling element is calculated according to equation (4). In the analysis, rollers are substituted by 1D springs. Each spring has a stiffness covering the roller itself and its contacts with both races.

$$Q_{jk} = c_s \delta_{j,k}^{10/9} \tag{4}$$

The spring stiffness $c_L$ distributed between laminas becomes $c_s$. For rollers and races made of steel $c_L$ is expressed in eq. (5), where Lwe (mm) is length of body and $n_s$ is number of laminas. Other possible values of $c_L$ (N / mm) can be found in the literature (Harris & Kotzalas, 2006). Exact value for considered geometry and materials can be also obtained by means FEM analysis.

$$cs = \frac{c_L}{n_s} = \frac{35948 \cdot L_{WE}^{8/9}}{n_s} \tag{5}$$

Last step of an analysis is an iterative solution of static force and momentum equilibrium equations (6) and (7) for $\delta_r$ (mm) and $\psi_j$ (rad) respectively.

$$Fr - cs \cdot \sum_{j=1}^{Z} \left( \cos(\varphi_J) \sum_{k=1}^{ns} \delta_{j,k}^{10/9} \right) = 0 \tag{6}$$

$$M_Z - cs \cdot \sum_{j=1}^{Z} \left( \cos(\varphi_J) \sum_{k=1}^{ns} x_k \cdot \delta_{j,k}^{10/9} \right) = 0 \tag{7}$$

**Contact Pressure calculation**

Contact pressure distribution on rollers was based on Hertz contact theory for line contact (Harris & Kotzalas, 2006). For each lamina and roller respectively, contact half width b (mm) and contact pressure $P_{jk}$ (MPa) were calculated according to equations (8) and (9), where R and E are defined by eq. (14) and (15); $w_s$ is length of lamina.

$$b_{jk} = \sqrt{\frac{8 \cdot (Q_{jk} / w_s) \cdot R_x}{\pi \cdot E'}} \tag{8}$$

$$P_{jk} = \frac{2 \cdot (Q_{jk} / w_s) \cdot R_x}{\pi \cdot b_{jk}} \tag{9}$$

**Lubrication layer thickness**

Minimal thickness of lubrication layer is calculated from equation (10) published by Dawson and Higginson. U, G and W are dimensionless groups; $R_x$ (mm) is mutual curvature of bodies in contact, perpendicular to rolling direction. Radius Ry (mm) for roller bearing with profile modification could be estimated by circle curve fitting to profile modification. h0 (mm) is minimal lubrication layer thickness in contact (Wheeler, et al., 2016). Following equation is modified for wide elliptical contacts, where Rx / Ry > 3. From asymptotic analysis it could be proved, that the last bracket with exponent in (10) becomes 1 for Rx / Ry > 20

$$h_0 = 3.63 \cdot R_y \cdot \overline{U}^{0.68} \cdot \overline{G}^{0.49} \cdot \overline{W}^{-0.073} \cdot (1 - e^{-0.70(Rx/Ry)^{0.64}}) \tag{10}$$

Equation (10) was obtained by curve fitting of numerical solutions of Reynolds equation for elliptical contact in wide range of conditions. Presented form is modified for wide contacts with ellipticity parameter Rx / Ry > 2.5. For details, see (Wheeler, et al., 2016). Main assumptions for such equation are: model of smooth contact, fully flooded lubrication regime, contact pressure maintained below 2 GPa and isothermal conditions within lubrication layer. It also assumes Newtonian behavior of lubricant. Wheeler et. al found and described in study (Wheeler, et al., 2016), that such semi-analytical equation, tends to overestimate minimal lubrication layer thickness about a 6 % comparing to full numerical solution. Therefore, in the model, minimal thickness was reduced by 6 %. In addition, most semi-analytical equations tends to deviate in operational conditions where high loads and low speeds occurs. Therefore, it is necessary to pay attention when assessing a lubrication regimes laying in areas where transition from boundary lubrication to EHD comes about. For accurate results, full numerical solution of Reynolds equation is recommended (Wheeler, et al., 2016). The application of (10) for line contact is further limited to pure radial loading or very light off-axis moment loading that does not considerably influence the load distribution over the length of element. For one-sided skewed load distributions on the roller, lubrication layer height would have to be verified by complete numerical model of contact. Dimensionless groups used in equations consist of following parameters:

Speed parameters: $\overline{U} = \dfrac{U \cdot \eta_0}{E' R_y}$ (11)

Material parameters: $\overline{G} = \alpha^* \cdot E'$ (12)

Load parameters: $\overline{W} = \dfrac{W^\backslash}{E' R_y^2}$ (13)

$R_x$ (mm) is mutual curvature of bodies in: $\dfrac{1}{R_y} = \dfrac{1}{R_{1y}} + \dfrac{1}{R_{2y}}$ (14)

E' (N/mm$^2$) is a mutual elastic modulus: $\dfrac{2}{E'} = \dfrac{1-\nu_1^2}{E_1} + \dfrac{1-\nu_2^2}{E_2}$ (15)

Where, $\eta_0$ is a lubricant dynamic viscosity at atmospheric pressure (MPa.s); $\alpha^*$ is a viscosity-pressure coefficient (1/MPa),

U is entraining speed of lubricant to the contact: $U = \dfrac{u_1 + u_2}{2}$ (mm/s) (16)

For roller bearing, with assumption of pure rolling, entraining speed could be calculated from kinematic analysis (Spikes, 2015). The cage/retainer revolution speed, the unknown, is subtracted from the rotation of inner and outer ring, so the rolling elements remain steady state and only rotate around their axis. This transformation allows writing two equations for roller peripheral speeds: at inner race and outer race. The unknown retainer speed is then removed by combination of equations and the peripheral speed of roller can be easily calculated. Entraining speed (16) is for roller bearing calculated acc.to (18)

$$U = \frac{R_i \omega_i}{2} \left( \frac{R_i + Dwe}{R_i + Dwe/2} \right) \quad (17)$$

**Lambda parameter**

Generally, in roller bearings, lubrication film thickness is of similar order as surface roughness. Therefore there is defined a lambda parameter, that compares minimal lubrication thickness to composite surface roughness. This is provided by equation (18), where $s_x^2$ is root mean square (RMS) value of roller and race surface roughness, h0 is minimal lubrication layer thickness calculated according to (10).The RMS value $s_x$ can be estimated from Ra surface parameter as 1.25Ra.

$$\Lambda = \frac{h_0}{\sqrt{s_{race}^2 + s_{roller}^2}} \quad (18)$$

Based on extensive experimental research following values of lambda were found to correspond with lubrication regimes: $\Lambda < 1$ the load is supported mainly by surface asperities, both bodies are in direct contact, asperities are deformed, surfaces are loaded by shear stress if sliding occurs. For $1 < \Lambda < 3$ the mixed lubrication regime is present, meaning that applied load is yet partly supported by lubricant thin film, but still direct contact between bodies exist. When $\Lambda > 3$, the lubrication layer separates both contact bodies, the load is supported only by lubricant and local deformation of the bodies influence the lubrication layer thickness. The load is except minor traction shear mainly in normal direction (Harris & Kotzalas, 2006).

## RESULTS AND DISCUSSION

**Bearing model**

The verification of bearing model was based on comparison results provided by software KISSsoft (KISSsoft, 2013) accompanied by a module for bearing calculation according to standard ISO (2008).

For a purpose of verification, a radial roller bearing N306 was used. The geometry is summarized in the **Tab 1**.

**Tab. 1** Summary of N306 parameters

| Parameter | Designation | Value | Dimension |
|---|---|---|---|
| Inner bore diameter | d | 30 | mm |
| Outer race diameter | D | 72 | mm |
| Width of bearing | B | 19 | mm |
| Number of rollers | Z | 12 | |
| Roller element diameter | Dwe | 11 | mm |
| Roller element active width | Lwe | 11 | mm |
| Inner ring race diameter | Di | 43,5 | mm |
| Radial internal clearance (SKF Group, 2013) | Pd | 0,0325 | mm |
| Basic dynamic load rating | C | 58 500 | N |
| Young modulus | E | 210 000 | $N/mm^2$ |
| Poisson ratio | ν | 0,3 | |

The verification of radial deflection and moment load was done for three radial load cases defined as P/C = 0,05; 0,1 and 0,2. Light moment loading was also applied, in order to have same load case as defined in KISSsoft for tested case. As you can see in Tab. 2 the deviation between radial deflection results obtained by the code implemented in this paper and the results obtained by KISSsoft was below 0.1 %. Similar situation is for misalignment assessment, where the error topped 6 %. The error might be introduced by different considered roller geometry in the KISSsoft calculation that would influence the angle.

**Tab. 2** Bearing model verification – radial and angular displacement

| P/C | Radial load [N] | Moment load [Nm] | Current model [μm] | KISSsoft [μm] | \|Error\| [%] | Current model [mrad] | KISSsoft [mrad] | \|Error\| [%] |
|---|---|---|---|---|---|---|---|---|
| 0,05 | 2 925 N | 0.05 | 28,226 | 28,222 | 0.015 | 0.017 | 0.017 | 0 |
| 0,1 | 5 850 | 0.13 | 34,73 | 34,731 | 0.003 | 0.032 | 0.034 | 6 |
| 0,2 | 11 700 | 0.35 | 45,198 | 45,203 | 0.011 | 0.066 | 0.069 | 4 |

Results of load distribution verification are stated in Tab. 3. Due to symmetrical loading of elements, only half of loaded elements were compared. Actual distribution is depicted in a Fig .2. Pressure distribution per lamina on the maximally loaded roller is depicted on the Fig. 3, numerical values of maximal pressure on inner race and outer race are evaluated in Tab. 4 and compared with results obtained from KISSsoft as verification. Absolute error indicates almost perfect fit.

**Tab. 3** Bearing load distribution verification
Load P/C=0.2

| Roller #No | Current model [N] | KISSsoft [N] | \|Error\| [%] |
|---|---|---|---|
| 5 | 576 | 577 | 0.17 |
| 6 | 3592 | 3590 | 0.05 |
| 7 | 4901 | 4905 | 0.08 |

**Tab. 4** Max contact pressure verification
Load P/C=0.2

| Property | Current model [N/mm$^2$] | KISSsoft [μm] | \|Error\| [%] |
|---|---|---|---|
| Inner race | 2140 | 2143 | 0.13 |
| Outer race | 1719 | 1721 | 0.12 |

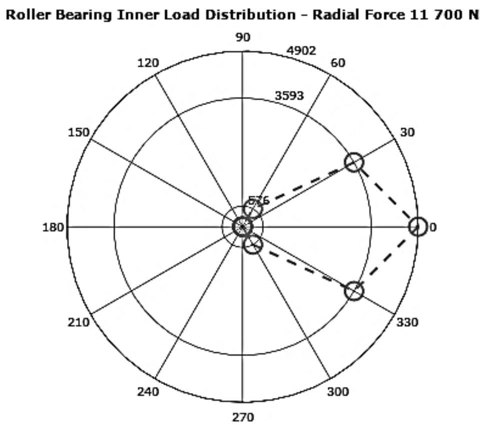


**Fig**. **2** Roller load distribution

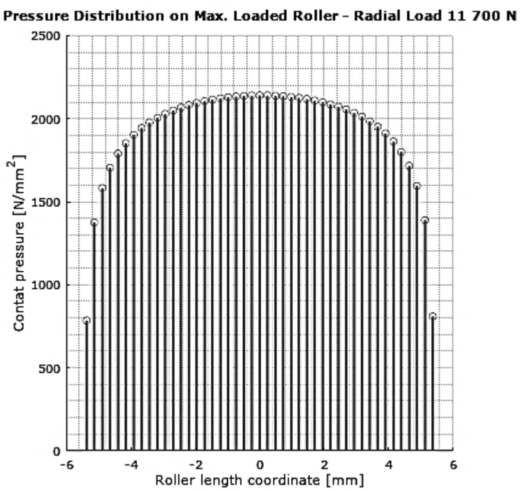


**Fig**. **3** Roller contact pressure distribution

**Lubrication**

Lambda parameter is calculated for every element – race contact. The result is graphically presented on the Fig.4. It is obvious, that less favourable lubrication conditions occur on the inner race contact, where lambda parameter for speed 2500 rpm indicates boundary lubrication $\Lambda = 2.2$. Lubrication regime on the outer race is analysed in the boundary regime too, but due to lighter loads, the lambda for most loaded element reaches $\Lambda = 2.8$.

**Tab. 5** Summary of parameters used for lubrication calculation

| Parameter | Designation | Value | |
|---|---|---|---|
| Lubricant dynamic viscosity at atm. pressure | $\eta_0$ (60°C) | $32.10^{-9}$ (32) | MPa.s (cP) |
| Viscossity-pressure coefficient | $\alpha^*$ (60°C) | 17 $.10^{-3}$ | $MPa^{-1}$ |
| Race (inner / outer) surface roughness | Ra | 0,08 | μm |
| Roller surface roughness | Ra | 0,03 | μm |
| Equivalent radius of profile modification | Ry | 1414 | mm |
| Bearing inner race speed | n | 2500 | rpm |

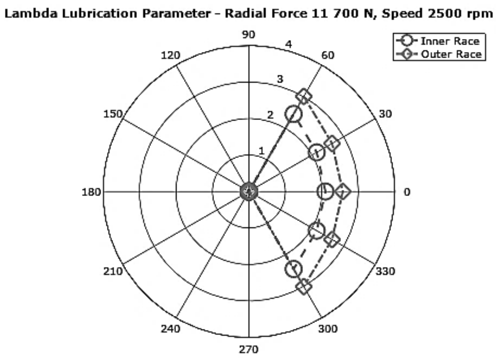


**Fig. 4** Lambda parameter distribution

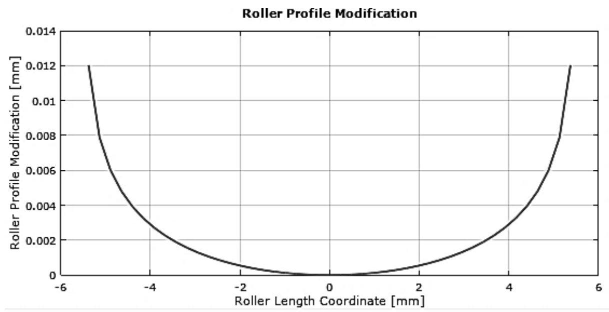


**Fig. 5** Roller profile modification

## CONCLUSIONS

Paper presents a mathematical model of a roller element bearing based on a method described in the standard (ISO, 2008) extended by calculation of lubrication layer thickness for every loaded element. Model calculates load distribution on rollers, contact pressure distribution on the roller. It is possible to apply a radial and moment load. Model was successfully verified against commercial CAE software KISSsoft. Lubrication is calculated according to semi-analytic equation for wide elliptical contact presented by Hamrock and Dowson (10). Lambda parameter (18) for estimation of lubrication regime by comparing lubrication layer thickness with surface composite roughness is calculated and results are presented as polar figure Fig 4. Lubrication calculation is limited by symmetrical load distribution on the roller element. Presented model will be further extended to cover other operating conditions.

## REFERENCES

1. International Organizatipon for Standardization. (2008). Rolling bearings – Methods for calculating the modified reference rating life for universally loaded bearings (ISO/TS 16281:2008).
2. Harris, T. A., & Kotzalas, M. N. (2006). Essential concepts of bearing technology. CRC press.
3. Wheeler, J. D., Vergne, P., Fillot, N., & Philippon, D. (2016). On the relevance of analytical film thickness EHD equations for isothermal point contacts: Qualitative or quantitative predictions?. Friction, 4(4), 369.
4. Spikes, H. (2015). Basics of EHL for practical application. Lubrication Science, 27(1), 45-67.
5. KISSsoft, A. G. (2013). KISSSoft Release 03/2013 User Manual. Bubikon, Switzerland.
6. SKF Group, (2013) Rolling Bearings, Catalogue, S. G.. PUB BU/P1 10000/2 EN. SKF.

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