METHOD OF DETERMINING OPTIMAL CONTROL STRATEGY

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Abstract
The paper presents a description of the method of determining the optimal (quasi-optimal) strategy of technical objects operation and maintenance process with the implementation of decisive semi-Markov processes as well as multicriteria genetic algorithm in which the result constitutes the set of optimal solutions according to Pareto (the so-called Pareto frontier). The method discussed in the paper allows for determining the quasi-optimal strategy of technical objects control process from the point of view of the values of selected criteria functions: technical objects availability as well as unit income generated during the carrying out of the analyzed operation and maintenance process. This is connected with selection from possible decisive variants of such strategy of process operation control for which the functions constituting evaluation criteria reach values from the set of Pareto optimal solutions.

Key words: *multicriteria optimization; genetic algorithm; decision processes; Pareto front.*

INTRODUCTION

The paper discusses the issues connected with the process of operation of technical objects. In complex technical objects operation systems, the selection of rational control decisions from possible decisive variants is should be carried out with the implementation of appropriate methods and mathematical tools rather than "intuitively" based solely on the knowledge and experience of the deciders of the systems. Introducing appropriate mathematical methods of operation process control facilitates selecting rational control decisions in a way which provides correct and effective carrying out of tasks assigned to the system. Depending on the kind of analyzed research problems, appropriate methods of delineating optimal and quasi-optimal solutions were implemented (e.g.: *Grabski, 2010; Knopik, Migawa & Wdzięczny, 2016; Kulkarni, 1995; Lee, 2000; Zastempowski & Bochat, 2014*). Genetic algorithm belongs to the group of nondeterministic methods of defining optimal solution in which consecutive solutions are random modifications of previous ones and significantly depend on them. The basic assumption while using genetic algorithm to search for optimal solution is the fact originating in the theory of evolution claiming that the smallest probability of modification is connected with solutions of the highest degree of adaptability defined as the value of adaptability function (function of the goal of optimizing task). The single criterion concept of genetic algorithm was developed by Holland in the 1960s, whereas the first practical methods were created at the turn of the 1960s and 1970s (*Holland, 1975*). The first genetic algorithm used in solving the questions of multiple criteria optimization of presented by Schaffer in 1985 and was called Vector Evaluated Genetic Algorithm (VEGA) (*Schaffer, 1985*). The following years saw an intense development of nondeterministic methods created on the basis of evolutionary algorithms. Using these methods made it possible to arrive at solutions in complicated issues of multi criteria optimization in a simple and fast way. The most important methods include (*Konak, Coit & Smith, 2006*): Multi-objective Genetic Algorithm (MOGA), Niched Pareto Genetic Algorithm (NPGA), Weight-Based Genetic Algorithm (WBGA), Random Weighted Genetic Algorithm (RWGA), Nondominated Sorting Genetic Algorithm (NSGA), Strength Pareto Evolutionary Algorithm (SPEA), Pareto-Archived Evolution Strategy (PAES), Pareto Envelope-based Selection Algorithm (PESA), Region-based Selection in Evolutionary Multiobjective Optimization (PESA-II), Fast Nondominated Sorting Genetic Algorithm (NSGA-II), Multi-objective Evolutionary Algorithm (MEA), Rank-Density Based Genetic Algorithm (RDGA) and Dynamic Multi-objective Evolutionary Algorithm (DMOEA). The objective of the paper is development of a method of determining technical objects operation process control strategy with the implementation of stochastic decisive models as well as nondeterministic methods of multi criteria optimization. The paper discusses the example of determining optimal control strategy in the case when criteria functions constitute the availability of technical objects (means of transport) as well as unit income (cost) generated while carrying out the analyzed operation process.

MATERIALS AND METHODS

Due to the random nature of the factors influencing the running of the technical objects (transport means) operation process introduced in a complex system, most often in the process mathematical modelling of the operation process, stochastic processes are used (Markov and semi-Markov processes as well as decision-making Markov and semi-Markov processes).

Assuming that the analyzed model of technical object operation process is a random process $\{X(t): t \geq 0\}$ of finite number of process states $i \in S = \{1,2,\ldots,m\}$, then

$$D_i = \left\{d_i^{(1)}(t_n), d_i^{(2)}(t_n), \ldots, d_i^{(k)}(t_n)\right\} \tag{1}$$

means a set of all possible control decisions which can be implemented in *i*-state of the process at the moment of t_n, where $d_i^{(k)}(t_n)$ means *k*-control decision made in *i*-state of the process, at the moment of t_n.

In the case of optimization task involving the choice of optimal strategy of technical object operation process control from among the acceptable strategies, then as the strategy we understand the δ sequence, where the words are the vectors, comprising of the decision $d_i^{(k)}(t_n)$ made in the following moments of the t_n changes of the state of the process $X(t)$

$$\delta = \left\{\left[d_1^{(k)}(t_n), d_2^{(k)}(t_n), \ldots, d_m^{(k)}(t_n)\right]: \; n = 0,1,2,\ldots\right\} \tag{2}$$

In order to determine the optimal control strategy (decision sequence) it is possible to implement decision-making semi-Markov processes. The decisive semi-Markov process is a stochastic process $\{X(t): t \geq 0\}$, the implementation of which depends on the decisions made at the beginning of the process t_0 and at the moments of changing the process $t_1, t_2, \ldots, t_n, \ldots$. At work it is assumed that the analyzed semi-Markov process possess a limited number of states $i = 1,2,\ldots,m$. In case of implementation of the decisive semi-Markov processes making the decision at the moment of t_n, *k*-controlling decision in *i*- state of the process means a choice of *i*-verse of the core of the matrix from the following set

$$\left\{Q_{ij}^{(k)}(t): t \geq 0, \;\; d_i^{(k)}(t_n) \in D_i, \;\; i, j \in S\right\} \tag{3}$$

where

$$Q_{ij}^{(k)}(t) = p_{ij}^{(k)} \cdot F_{ij}^{(k)}(t) \tag{4}$$

The choice of the *i*-verse of the core of the process specifies the probabilistic mechanism of evolution of the process in the period of time $<t_n, t_{n+1})$. This means that for the semi-Markov process, in case of the change of the state of the process from one into *i*-one (entry to the *i*-state of the process) at the moment t_n, the decision is made $d_i^{(k)}(t_n) \in D_i$ and according to the schedule $\left(p_{ij}^{(k)} : j \in S\right)$ *j*-state of the process is generated, which is entered at the moment of t_{n+1}. At the same time, in accordance with the schedule specified by the distributor $F_{ij}^{(k)}(t)$, the length of the period of time is generated $<t_n, t_{n+1})$ to leave the *i*-state of the process, when the next state is the *j*-state. The choice of appropriate control strategy δ called the optimal strategy δ, concerns the situation, when the function (functions) representing the selection criterion of the optimal strategy takes an extrene value (minumum or maximum)

$$f_C\left(\delta^*\right) = \min_{\delta}\left[f_C(\delta)\right] \;\; or \;\; f_C\left(\delta^*\right) = \max_{\delta}\left[f_C(\delta)\right] \tag{5}$$

In the paper, the criteria functions are the availability of individual technical object $A^{OT}(\delta)$ and the unit income generated in the states of the modeled operation and maintenance process $C^{OT}(\delta)$:

$$f_{C_1}(\delta)=A^{OT}(\delta)=\sum_{i\in S_A} p_i^*(\delta)=\frac{\sum_{i\in S_A}\pi_i\cdot\Theta_i(\delta)}{\sum_{i\in S}\pi_i\cdot\Theta_i(\delta)} \tag{6}$$

$$f_{C_2}(\delta)=C^{OT}(\delta)=\sum_{i\in S} c_i(\delta)\cdot p_i^*(\delta)=\frac{\sum_{i\in S} c_i(\delta)\cdot\pi_i\cdot\Theta_i(\delta)}{\sum_{i\in S}\pi_i\cdot\Theta_i(\delta)} \tag{7}$$

where:

$S_A \subset S$ – set of availability states of modeled operation and maintenance process,

$c_i(\delta)$ – unit incomes generated in the states of process $X(t)$,

$p_i^*(\delta)$ – limit probabilities of remaining in states of the analyzed process $X(t)$ were determined based on limit theorem for semi-Markov processes (*Grabski, 2014*)

$$p_i^*(\delta)=\frac{\pi_i\cdot\Theta_i(\delta)}{\sum_{i\in S}\pi_i\cdot\Theta_i(\delta)} \tag{8}$$

where:

$\Theta_i(\delta)$ – average values of unconditional duration of the states of process,

π_i – probabilities of stationary distribution of the complex Markov chain fulfilling the system of linear equations

$$\sum_{i\in S}\pi_i\cdot p_{ij}=\pi_j,\quad j\in S,\quad \sum_{i\in S}\pi_i=1 \tag{9}$$

p_{ij} – conditional probability of passing from state i to state j:

$$p_{ij}=\lim_{t\to\infty} p_{ij}(t) \tag{10}$$

$$p_{ij}(t)=P\{X(t)=j|X(0)=i\} \tag{11}$$

The choice of the optimal strategy δ^* is made on the basis of the following criterions:

$$A^{OT}(\delta^*)=\max_{\delta}\left[A^{OT}(\delta)\right],\quad C^{OT}(\delta^*)=\max_{\delta}\left[C^{OT}(\delta)\right] \tag{12}$$

The genetic algorithm constitutes the convenient tool for selection the optimal strategy δ^* process control operation of technical objects on the base of developed semi-Markov model of the process. In case of the implementation of the genetic algorithm to determine the optima strategy of controlling the operation processes for technical objects, the following guidelines should be considered:

– the examined stochastic process is the m-state decisive semi-Markov process,
– in each state it is possible to implement one of the two decision $D = \{0,1\}$,
– if the decisions are marked as 0 and 1 then the number of control strategies to be implemented for the m-state model of the operation process of the means of transport amounts to 2^m,
– the set of control strategies is the set of functions $\delta : S \to D$.

On the basis of the following guidelines each possible control strategy can be presented as m-positioning sequence consisting of 0 and 1. Therefore, an exemplary control strategy for the model of the operation process consisting of m = 9 states can be determined in the following way: $\delta=[1,0,1,1,0,1,0,0,1]$. For 9th state semi-Markov model of the means of transport operation and maintenance process presented in the paper (*Migawa, Knopik & Wawrzyniak, 2016*) as well as data obtained from tests of the existing operation and maintenance system, calculations were made with the help of developed computer software, implemented multicriteria genetic algorithm.

RESULTS AND DISCUSSION

The presented example was prepared on the basis of operational data obtained from tests of the existing means of transport operation system (municipal transport buses). In the tested system 182 municipal buses are in use, while the service and repair processes are carried out at technical infrastructure posts as well as technical emergency units. Operation process control is possible as a result of correct decision making at decisive states of the process (Tab. 1). The analyzed model of the operation and maintenance

process distinguishes the following states of the technical object: 1 – awaiting at the bus depot parking space, 2 – preparation at the bus depot parking space during the stanby time, 3 – carrying out of the transport task, 4 – refuelling between of the transport task, 5 – diagnosing and repair by the technical support unit without losing a ride, 6 – diagnosing and repair by technical support unit with losing a ride, 7 – awaiting the start of task realization after repair by technical support unit, 8 – emergency exit, 9 – realization of maintenance process at the posts of serviceability assurance subsystem (refuelling, check on the operation day, realization of periodical servicing, diagnosing, repair).
For the analyzed model of the operation and maintenance process of means of transport values of input parameters of genetic algorithm were determined, while, on the basis of operation data, the values of the elements of probability transfer matrix (13) were determined, as well as possible decisions made at decisive states of the process (Tab. 1) and mean values of time periods as well as unit incomes generated at the states of the process were established (Tab. 2).

$$P = \begin{bmatrix} 0 & 0.02831 & 0.97169 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.14377 & 0.08818 & 0.03520 & 0 & 0.03753 & 0.69532 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.10224 & 0 & 0 & 0 & 0.89776 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (13)$$

Tab. 1 Decisions at states of analyzed process $X(t)$

Process state	Decision „0” - $d_i^{(0)}$	Decision „1” - $d_i^{(1)}$
1	Not decision-making process state	
2	Preparation type N (normal)	Preparation type I (intensive)
3	The route marked code L („light” conditions of the delivery task)	The route marked code D („difficult” conditions of the delivery task)
4	Not decision-making process state	
5	Repair type B (basic range)	Repair type E (extended range)
6	Repair type B (basic range)	Repair type E (extended range)
7	Not decision-making process state	
4	Not decision-making process state	
9	Maintenance process type N (normal)	Maintenance process type I (intensive)

Tab. 2 Mean time periods and unit incomes generated at states of process $X(t)$

Process state	$\Theta_i^{(0)}$ [h]	$\Theta_i^{(1)}$ [h]	$c_i^{(0)}$ [PLN/h]	$c_i^{(1)}$ [PLN/h]
1	5.659	5.659	-6.77	-6.77
2	0.280	0.224	-68.08	-87.21
3	8.852	7.967	28.44	33.56
4	0.743	0.743	-8.88	-8.88
5	0.336	0.318	-53.28	-68.86
6	0.811	0.691	-101.87	-132.43
7	0.442	0.442	-24.66	-24.66
8	0.712	0.712	-231.70	-231.70
9	2.064	1.880	-48.02	-55.01

Genetic algorithm input parameter values: length of chromosome $m = 9$, size of population $n = 100$, number of iterations $I = 100$, probability of chromosome selection via elitism principle $\eta = 0.2$, probability of hybridization occurrence $\kappa = 1$, probability of mutation occurrence $\mu = 0.05$.
In the presented model the states of the availability of technical objects are: 1, 2, 3, 7.
Then, calculations were performed with the help of the developed computer program implementing the multicriteria genetic algorithm. On the basis of the calculations performed, for the criteria adopted, optimal (quasi-optimal) control strategies were determined for the operation and maintenance process carried out at the tested transport system. Calculation results were presented in Fig. 1 as well as Tab. 3.

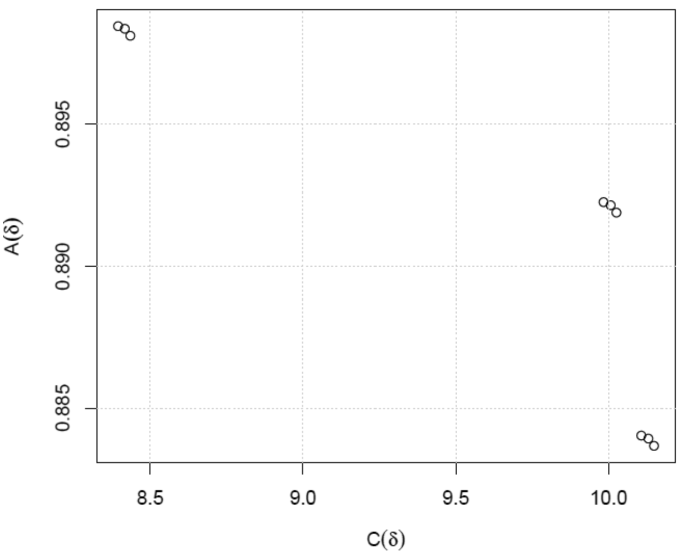

Fig. 1 Pareto frontier for optimal solutions determined on the basis of multicriteria genetic algorithm

Tab. 3 Optimal control strategies δ^* as well as value of criteria functions determined on the basis of multicriteria genetic algorithm

Strategy δ^*	$A^{OT}(\delta^*)$	$C^{OT}(\delta^*)$ [PLN/h]
[1,0,0,0,1,1,0,0,1]	0.8984	8.39
[0,0,0,1,0,1,0,0,1]	0.8983	8.42
[1,0,0,1,0,0,1,0,1]	0.8981	8.44
[0,0,1,0,1,1,0,0,1]	0.8922	9.98
[1,1,1,1,0,1,1,1,1]	0.8921	10.01
[1,1,1,1,0,0,0,1,1]	0.8919	10.02
[0,0,1,1,1,1,1,0,0]	0.8840	10.11
[0,1,1,0,0,1,0,0,0]	0.8839	10.13
[1,1,1,0,0,0,1,0,0]	0.8837	10.15

Articles (*Migawa, Knopik & Wawrzyniak, 2016*) and (*Migawa, Knopik, Neubauer & Perczyński, 2017*) discuss 9-state models of the means of transport use process (semi-Markov and simulation process, respectively) the implementation of which facilitates determining a control strategy taking into consideration a single criterion (availability of technical objects). The method presented in the article, on the other hand, applies to multi-criteria analysis (e.g. when two criteria of evaluation are applied, such as availability and unit income). On the basis of the results obtained it is noticeable that an increase in means of transport availability is connected to lowering of unit income. This stems mainly from the need to bear additional costs connected to maintaining the roadworthiness of technical objects. This is carried out by an increase of intensity of services and repairs performed, e.g. involving more efficient tools and devices as well as a bigger number of workers. In the example given, one may notice that the increase in availability from the level of ca. 0.884 to 0.892 is possible with little additional expenses (a decrease of income from ca. 10.13 to 10.00 [PLN/h]). However, a further increase in availability of technical objects above the value of 0.898 requires a significant increase of expenses on services and repairs of technical objects, thus resulting in a decrease of unit income below the level of 8.45 [PLN/h].

CONCLUSIONS

On the basis of the results of operation tests at the existing system of means of transport operation, input data were determined for the developed genetic algorithm and calculations were performed. As a result, the values of criteria functions as well as a corresponding set of control strategies constituting a set of optimal solutions according to Pareto were determined. The optimal set according to Pareto is a set of non-dominated solutions of the whole acceptable search space. Optimal solutions according to Pareto form the so-called Pareto frontier. On the basis of the results obtained, a selection of a single solution from the determined set of optimal solutions (located on the so-called Pareto frontier) may be made. Such selection is usually made by the decider (a group of deciders) on the basis of additional circumstances connected to particular decisive situation as well as current conditions in which the operation and maintenance system functions. Depending on the requirements, the developed genetic algorithm, including the decisive model of operation and maintenance process, may be used for mathematically formulating and solving a wide range of problems connected with control of complex technical systems. This is mainly connected to economical analysis, risk and safety managements as well as availability and reliability of the utilized technical objects.

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