TWO-MASS LINEAR VIBRATORY CONVEYOR

Martin STURM[1], Lubomír PEŠÍK[2]

[1]*Faculty of Mechanical Engineering, University of Applied Sciences Zittau/Görlitz, Germany*
[2]*Faculty of Mechanical Engineering, Department of Machines Parts and Mechanisms, Technical University of Liberec, Czech Republic*

Abstract
Linear vibratory conveyors are a common equipment for conveying goods in different industries such as for example building, mining, food industry. These systems are used for the supply of mainly bulk goods into further processing operations. The goods to be conveyed typically requesting a heavy duty design of the conveyor and eccentric excitation drives with relatively high torque, supplying strong vibrations to the floor of the installation location. The target of this article is to determine a dynamic model, based on a two-mass absorber system, for the effective avoidance of the transfer of vibrations to the ground by enabling an optimum supply of goods by the same time.

Key words: *Vibratory conveyor, dynamic absorber, resonance frequencies, dynamic model.*

INTRODUCTION
Linear vibratory conveyors are mainly built to forward bulk material for the mining, building and food industry, such as rock, grit and flour. The assembly of these vibratory conveyors consists of two principal parts, the conveying element build as a conveying trough and a supporting element, build of steel elements and fixed to the ground by heavy-duty screws (Harris, 2005), (Buja, 2007). This type of conveyors representing a one-mass system with the disadvantage to have only very limited possibilities to optimize the efficiency regarding the conveying of the goods and even more, the reduction of vibrations transmitted to the floor.
A two-mass conveyor as shown in Fig. 1 represents an alternative solution to the one-mass linear vibrating conveyor, which is not so common.

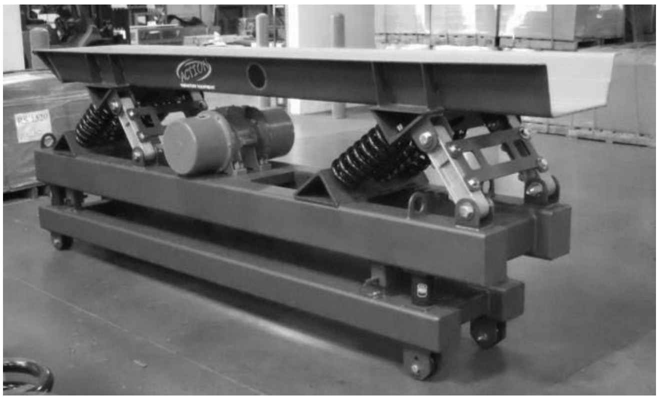

Fig. 1 Vibratory conveyor with two masses (*Action Equipment Company*, Inc., 2017)

MATERIALS AND METHODS
The theoretical solution is done in two steps. At first, a dynamic model is deigned and then the dynamic parameters are determined.

Determination of the conveyors dynamic model
The assembly of the following two-mass vibratory conveyor consists of three principal parts, the conveying element build as a conveying trough, represented by character m_2, the excited mass with the drive, represented by character m_1 and a supporting element, fixed to the ground. Both masses are connected by springs. In this case, helical springs are installed. The excitation of m_1 is done by an eccentric drive, installed to excite the mass 90° to the lever (Dresig, Holzweißig, 2008). The stiffness and the springs are defined as k_1 and k_{21}. The damping of the system is neglected. Mass $m_1$is con-

nected to the ground by levers under an angle of β_1, mass $m_2$is connected to mass $m_1$by levers under an angle of β_2. The levers enabling a better definition of the masses motion. Fig. 2 shows the mechanical model of the two-mass conveyor.

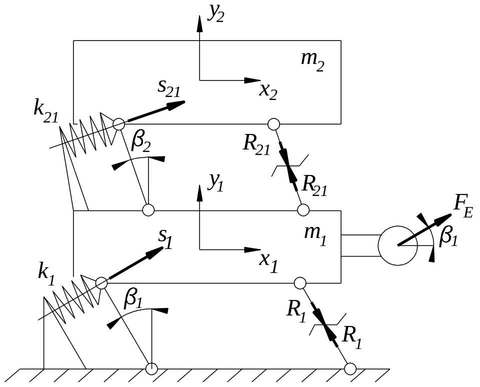

Fig. 2 Mechanical model of the vibratory conveyor

Calculation of dynamic parameters

Based on the mechanical model shown in Fig. 2, the following equations are formed (Knaebel, Jäger, Roland, 2009):

$$m_1\ddot{x}_1 + k_1x_1 - k_{21}x_{21} - F_{R1}\sin\beta_1 + F_{R21}\sin\beta_2 = F_E\cos\beta_1\ , \tag{1}$$

$$m_1\ddot{y}_1 + k_1y_1 - k_{21}y_{21} - F_{R1}\cos\beta_1 + F_{R21}\cos\beta_2 = F_E\sin\beta_1\ , \tag{2}$$

$$m_2\ddot{x}_2 + k_{21}x_{21} - F_{R21}\sin\beta_2 = 0\ , \tag{3}$$

$$m_2\ddot{y}_2 + k_{21}y_{21} - F_{R21}\cos\beta_2 = 0\ , \tag{4}$$

Summing up the equations (1) and (3), (2) and (4) as well as (3) and (4), the following equations can be formed:

$$m_1\ddot{x}_1 + k_1x_1 - F_{R1}\sin\beta_1 + m_2\ddot{x}_2 = F_E\cos\beta_1\ , \tag{5}$$

$$m_1\ddot{y}_1 + m_2\ddot{y}_2 + k_1y_1 - F_{R1}\cos\beta_1 = F_E\sin\beta_1\ , \tag{6}$$

$$(m_2\ddot{x}_2 + k_{21}x_{21})cos\beta_2 + (m_2\ddot{y}_2 + k_{21}y_{21})sin\beta_2 = 0\ . \tag{7}$$

Merging equation (5) and (6), the equation can be formed as

$$\frac{m_1\ddot{x}_1+k_1x_1+m_2\ddot{x}_2}{sin\beta_1} + \frac{m_1\ddot{y}_1+k_1y+m_2\ddot{y}_2}{cos\beta_1} = F_E\ \frac{\cos\beta_1}{\sin\beta_1} + F_E\ \frac{\sin\beta_1}{\cos\beta_1}, \tag{8}$$

and after converting equation (8)

$$(m_1\ddot{x}_1 + m_2\ddot{x}_2 + k_1x_1)\cos\beta_1 - (m_1\ddot{y}_1 + m_2\ddot{y}_2 + k_1y_1)\sin\beta_1 = F_E\ , \tag{9}$$

Splitting up the motion of x_2 and $y_2$into the components, the following form can be written

$$(m_1\ddot{x}_1 + m_2\ddot{x}_1 + m_2\ddot{x}_{21} + k_1x_1)\cos\beta_1 - (m_1\ddot{y}_1 + m_2\ddot{y}_2 + m_2\ddot{y}_{21} + k_1y_1)\sin\beta_1 = F_E\ , \tag{10}$$

$$(m_2\ddot{x}_1 + m_2\ddot{x}_{21} + k_{21}x_{21})cos\beta_2 + (m_2\ddot{y}_1 + m_2\ddot{y}_{21} + k_{21}y_{21})sin\beta_2 = 0\ . \tag{11}$$

The levers guide the motion of each of the two masses between floor and mass m_1 and between mass $m_1$and mass m_2. Therefore the coordinate vectors with the characters x and y can be merged and replaced by character s. Then the equations (10) and (11) can be formed as

$$m_1\ddot{s}_1\cos^2\beta_1 + m_2\ddot{s}_1\cos^2\beta_1 + m_2\ddot{s}_{21}\cos\beta_2\cos\beta_1 + k_1s_1\cos^2\beta_1 + m_1\ddot{s}_1\sin^2\beta_1 + m_2\ddot{s}_1\sin^2\beta_1 + \\ +m_2\ddot{s}_{21}\sin\beta_2\sin\beta_1 + k_1s_1\sin^2\beta_1 = F_E\ , \tag{12}$$

$$m_2\ddot{s}_1\cos\beta_1\cos\beta_2 + m_2\ddot{s}_{21}\cos^2\beta_2 + k_{21}s_{21}\cos^2\beta_2 + m_1\ddot{s}_1\sin\beta_1\sin\beta_2 + m_2\ddot{s}_{21}\sin^2\beta_2 + \\ +k_{21}s_{21}\sin^2\beta_2 = 0\,. \tag{13}$$

with

$$s_{21} = s_2 - s_1 \tag{14}$$

After simplifying the equations (12) and (13), the final equations of motion for the analyzed conveyor model are

$$(m_1 + m_2)\ddot{s}_1 + m_2\ddot{s}_{21}(\cos\beta_2\cos\beta_1 + \sin\beta_2\sin\beta_1) + k_1s_1 = F_E\,, \tag{15}$$

$$m_1\ddot{s}_1(\cos\beta_2\cos\beta_1 + \sin\beta_2\sin\beta_1) + m_2\ddot{s}_{21} + k_{21}s_{21} = 0\,. \tag{16}$$

RESULTS AND DISCUSSIONS

The results of the above equations can be used for the optimisation of dynamic parameters.

Variation of the angles of the dynamic model

For the further analysis of the dynamic model, three cases of particular interest have to be observed, i.e. case one with $\beta_1 = \beta_2$, case two with $\beta_1 = 0$ and case three with $\beta_2 = 0$.

For case one with $\beta_1 = \beta_2$, equation (15) can be converted step by step

$$(m_1 + m_2)\ddot{s}_1 + m_2\ddot{s}_{21} + k_1s_1 = F_E\,, \tag{17}$$

$$m_1\ddot{s}_1 + m_2\ddot{s}_1 + m_2\ddot{s}_2 - m_2\ddot{s}_1 + k_1s_1 = F_E\,, \tag{18}$$

$$m_1\ddot{s}_1 + m_2\ddot{s}_2 + k_1s_1 = F_E\,, \tag{19}$$

to the final form

$$m_1\ddot{s}_1 + k_1s_1 - k_{21}(s_2 - s_1) = F_E\,. \tag{20}$$

Equation (16) is transformed by the same way into the final form

$$m_2\ddot{s}_2 + k_{21}(s_2 - s_1) = 0. \tag{21}$$

For case two with $\beta_1 = 0$, the equations are

$$(m_1 + m_2)\ddot{s}_1 + m_2\ddot{s}_{21}cos\beta_2 + k_1s_1 = F_E\,, \tag{22}$$

$$m_2\ddot{s}_1cos\beta_2 + m_2\ddot{s}_{21} + k_{21}s_{21} = 0. \tag{23}$$

For case three with $\beta_2 = 0$, the following equations can be formed

$$(m_1 + m_2)\ddot{s}_1 + m_2\ddot{s}_{21}cos\beta_1 + k_1s_1 = F_E\,, \tag{24}$$

$$m_2\ddot{s}_1cos\beta_1 + m_2\ddot{s}_{21} + k_{21}s_{21} = 0. \tag{25}$$

Calculation of the displacements

The corresponding equations for case one, i.e. $\beta_1 = \beta_2$ to find the zero displacement s_{10} and the maximum displacement s_{20} of mass m_1 can be formed as

$$s_{10} = \frac{(k_{21} - m_1\omega^2)F_{E0}}{(k_1 - m_1\omega^2)(k_{21} - m_2\omega^2) - k_{21}m_2\omega^2}\,, \tag{26}$$

$$s_{20} = \frac{k_{21}F_{E0}}{(k_1 - m_1\omega^2)(k_{21} - m_2\omega^2) - k_{21}m_2\omega^2}\,. \tag{27}$$

Equation (26) can be adjusted in a way that s_{10} will become zero. For that case, the minimum transmission of forces to the floor is achieved (Nendel, 2008), (Risch, 2008).

Based on this dynamical model, the optimum parameters for an efficient conveying of goods by a minimum transmission of vibration and forces into the floor can be found.

The optimum conveying of goods and the minimum transmission of vibration by the same time is given for the case that mass m_1 during the excitation remains stationary, while the full excitation is passed over to the absorber mass m_2 (Pešík, 2013).

CONCLUSION

The paper deals with the determination of dynamic model parameters of a two-mass linear vibratory conveyor conducted with the calculation of kinematic parameters for the model. The result of the elaboration is a mechanical model, which can be used as base for the modification of the conveyors motion and simultaneous the elimination of vibration forces to the floor of a building. The excitation of the system is connected to an absorber mass which supplies the full effect to the motion of the goods to be conveyed, without the transfer of vibrations to the ground.

General equations for the model are developed and the equations for the calcuation of minimum and maximum displacements of the excited mass are shown.

Based on the results of the paper, parameters for the design of two-mass linear vibratory conveyors can be defined and simultated.

AKNOWLEDGEMENTS

This publication was written at the Technical University of Liberec as part of the project "Innovation of products and equipment in engineering practice" with the support of the Specific University Research Grant, as provided by the Ministry of Education, Youth and Sports of the Czech Republic in the year 2017.

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Corresponding author:

M.A. Dipl.-Ing. Martin Sturm, HS Zittau/Görlitz Faculty of Mechanical Engineering, Germany, e-mail: m.sturm@hszg.de