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KINEMATIC SIMILARITY OF ROLLING BEARINGS WITH PLANETARY GEARS – A REVIEW

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Abstract
The knowledge of the bearing kinematics is very important for their diagnostics. There are more methods to determine the number of revolutions of all bearing components. It is necessary to know their values for the solving of diagnostic problems. The first method is based on the same tangent velocity of contact points of the bodies. The second one is much more transparent and is based on theory of planetary gears.

Key words: *bearing diagnostics, bearings kinematics, bearings damages, slowly running bearings.*

INTRODUCTION

The kinematics of rolling bearings (Fig. 1) falls into the area concerning simultaneous planar motions of bodies.

Fig. 1 Rolling bearings (ball and roller bearings)

They are described in elementary form by basic motion 21 and relative motion 32. The basic motion means the motion of carrier 2 relative to frame 1, and the relative motion means the motion of body 3 relative to carrier 2. In the plane, there are four combinations of elementary simultaneous motions of bodies 3 (translation-translation, translation-rotation, rotation-translation and rotation-rotation).
Kinematics of rolling bearings, in this sense, falls into the area of two simultaneous rotations.

KINEMATICS OF PLANETARY MECHANISM

In kinematics rolling, bearings represent a planetary gearing, in which the planet carrier is formed with a cage, and the rolling elements are planets (Fig. 2). This analogy allows the use of a variety of procedures well known from the theory of planetary gearing.
While carrier 2, wheels 4 and 5 with the central axis of rotation perform rotary motion, satellite 3 performs two simultaneous rotations around its axis and together with carrier 2 and rotation around the central axis of rotation.
For the description of kinematics of individual bodies bearing, it is possible to use methods that are used with kinematics of planetary gears. It is possible to use two methods, where with the character , there are described semi-diameters of several bearing components.
The first one is based on the conditions of rolling, and on equal peripheral velocity in respective contact points, which are the poles of relative motions.
The second method is based on substitution of planetary gear for countershaft gear. This substitution lies in conceptual stopping of the carrier. In such a case, an observer on the carrier can see how the gear rotates with relative velocity to the carrier. Then, the gear ratio can be expressed as the ratio of the relative angular velocity. Thus, the kinematic linkage of gearing is defined.

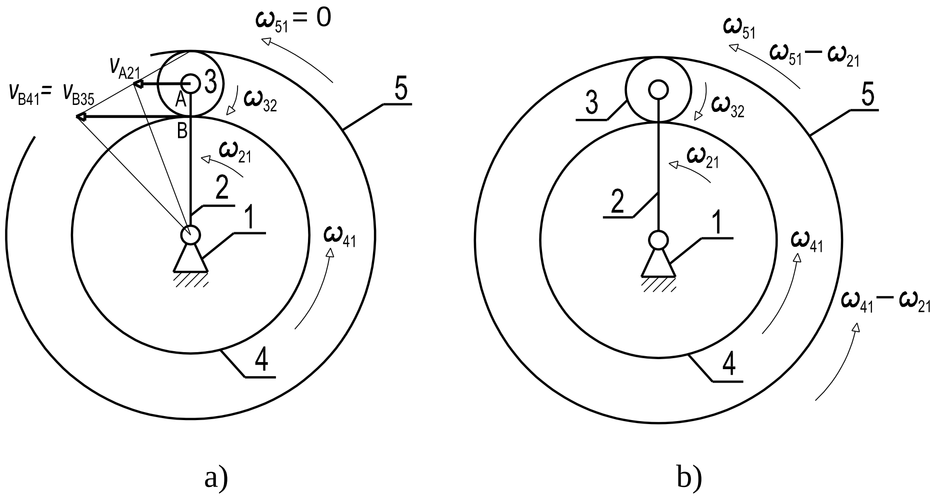

Fig. 2 Kinematics of planetary gearing with the help of rolling conditions

The angular velocity of any part of the gearing can be determined from such a linkage.
The first procedure is illustrated in Fig. 2a. When the gear 5 is not in motion and gear 4 is a driving gear, it applies that the tangential velocities

$$v_{B41} = v_{B35} \,. \tag{8}$$

Then

$$r_4\omega_{41} = 2r_3\omega_{35} \,. \tag{9}$$

Further

$$v_{A21} = \frac{v_{B41}}{2} \tag{10}$$

and from that

$$\omega_{21} = \frac{r_4}{2r_2}\omega_{41} = \frac{r_4}{r_4 + r_5}\omega_{41} \,. \tag{11}$$

To determine ω_{32}, basic decomposition can be used for velocity of point B

$$v_{B32} = v_{B35} + v_{B52} \tag{12}$$

And after substitution

$$r_3\omega_{32} = 2r_3\omega_{35} + (r_2 - r_3)\omega_{52} \,. \tag{13}$$

If we consider that

$$2r_3\omega_{35} = r_4\omega_{41} \tag{14}$$

And that ω_{52}, is

$$\omega_{52} = \omega_{51} - \omega_{21} \,. \tag{15}$$

We get

$$r_3\omega_{32} = r_4\omega_{41} - (r_2 - r_3)\omega_{21}. \tag{16}$$

And after the substitution of ω_{21}

$$r_3\omega_{32} = r_4\omega_{41} - (r_2 - r_3)\frac{r_4}{r_4 + r_5}\omega_{41}\ , \tag{17}$$

if

$$r_2 = r_4 + r_3 \tag{18}$$

we get for ω_{32}

$$\omega_{32} = \frac{r_5}{r_3}\frac{r_4}{r_4 + r_5}\omega_{41}\ , \tag{19}$$

which is the angular velocity of satellite 3 to carrier 2.
The method for calculating the angular velocity of planetary gear with the help of its substitution for countershaft gear is significantly shorter and the outcome more effective, especially in more complicated cases where the planetary gearing is combined. The essence of this method is shown again on an example of simple planetary gearing (Fig. 2b).
After definition of the positive senses of angular velocities of all parts of the gearing, which in this case is a differential (two degrees of freedom), we accede to the substitution of the planetary gearing for countershaft gearing, by imagining our position as an observer on the carrier. This way the carrier's motion towards us was stopped, although the kinematics of gears was not changed. The motion of the parts of the gearing is relative towards the carrier.
It is possible to compute the gear ratio of the relative angular velocity of any two parts of such defined countershaft mechanism. For example, the gear ratio of the relative angular velocity between gears 4 and 5 is

$$\frac{\omega_{41} - \omega_{21}}{\omega_{51} - \omega_{21}} = \frac{r_3}{r_4}\left(-\frac{r_5}{r_3}\right) = -\frac{r_5}{r_4}, \tag{20}$$

of which for $\omega_{51} = 0$ is

$$\omega_{21} = \frac{r_4}{r_4 + r_5}\ \omega_{41} \tag{21}$$

or similarly between gears 4 and 3 is,

$$\frac{\omega_{41} - \omega_{21}}{\omega_{32}} = \frac{r_3}{r_4} \tag{22}$$

and finally between gears 3 a 5 is

$$\frac{\omega_{32}}{\omega_{51} - \omega_{21}} = -\frac{r_5}{r_3}, \tag{23}$$

of which

$$\omega_{32} = \frac{r_5}{r_3}\frac{r_4}{r_4 + r_5}\omega_{41}\ . \tag{24}$$

It needs to be mentioned that angular velocity of planet 3, ω_{32}, is determined as relative to the carrier 2 considering that planet 3 has a different axis of rotation than the carrier and other parts of the planetary gearing.
If we use $\omega_{51} = 0$ in formulas (21) and (24) above, angular velocity ω_{21} of carrier 2 and angular velocity ω_{32} of planet 3 around the journal of the carrier 2 will be the same as in equations (11) and (19).
The planet gearing may be suitably combined to create gearing with the desired kinematic linkage. In some cases, double planets are used. However, in relation to the kinematics of the roller bearings it does not have adequate importance.

CONCLUSIONS

The kinematics of rolling bearing is necessary to be known for their diagnostics. The angular velocities determine frequencies in a frequency spectrum where damages of bearing parts can be monitored. The damage of the rolling body can be observed by the frequency dependent on angular velocity ω_{32}. The damage of the inner ring has its frequency which is given by number of rolling bodies and relative angular frequency $\omega_{41} - \omega_{21}$. The damage frequency of the outer ring is determined by number of rolling bodies and the angular velocity ω_{21}. The mentioned results are valid relative exact in the cases of slowly rolling bearings, but by fast running bearings, there it is necessary to calculate with slip between rolling bodies and inner or outer ring.

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REFERENCES

1. Beneš,Š., Tomeh, E. (1991). *Metody diagnistiky valivých ložisek.*
2. Pešík, L. (2015). *Části strojů, 1. díl*. Technická univerzita v Liberci.
3. Pešík, L. (2015). *Části strojů, 2. díl*. Technická univerzita v Liberci.
4. Helebrant, F., Ziegler, J.: *Technická diagnostika a spolehlivost II – Vibrodiagnostika.* VŠB – TU Ostrava, Ostrava 2004, 1. vydání, 178 s., ISBN 80 – 248 – 0650 – 9.
5. Randall, R. B. (1987) *Frequency Analysis*, Denmark: Brüel&Kjær.
6. Němeček, P., Tomeh, E. (2010). *Vibrační diagnostika základních závad strojů*. Technická univerzita v Liberci.
7. Tomeh, E. (2007). *Diagnostic Methodology of Rolling Element and Journal Bearings.* Technická univerzita v Liberci.ISBN 978-80-7372-278-4.

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