GEOMETRIC AND MESHING PARAMETERS OF A PINION-RACK PAIR – A REVIEW

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Abstract
This contribution deals with a relatively neglected area of meshing of a pinion-rack pair. Although it is in principle for a mesh between a pinion and a wheel with a very large number of teeth (infinitely one) so they can not apply all calculations of a pair pinion-wheel. This is mainly due the fact that they cannot work with neither central distances nor wheel diameters. Despite all relevant geometrical and meshing calculations are relatively easy ones.

Key words: *pinion; rack; geometry; mesh.*

INTRODUCTION

Pair pinion-rack is widespread in the area of gearing transmissions. As far as data for designers there is relatively well processed area of loading capacity calculations. But an area of geometric design is slightly neglected. Since a rack is a part of a wheel with infinitely big diameter, so there is this rack replaced by a wheel with a large number of teeth – for instance 1000 in geometric calculations. Results of such a mesh look like a lot to a mesh pinion-rack. But they are not fully exact. For achieving absolutely exact data it should be necessary to utilize equations which are commonly used for a design of a pair pinion-wheel. But they must be solved as limits for $z_2 \rightarrow \infty$. Some equations would be then unsolvable or they would became indeterminate expressions. This contribution shows some solving of these equations in the form right for toothed rack, otherwise $z_2 \rightarrow \infty$. Basic profile of the rack is on the fig. 1. On the fig. 2 is the basic profile of the tool for manufacturing of racks. It is a complex tool including for chamfering and for a protuberance undercutting of the rack.

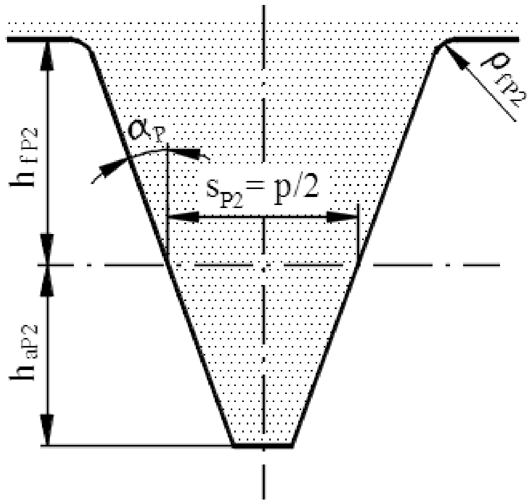


Fig. 1

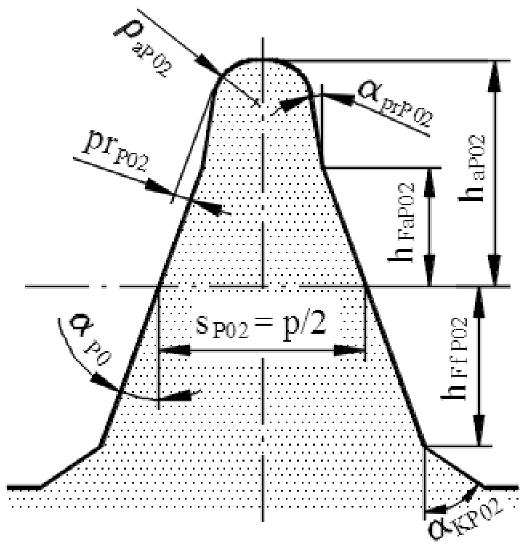
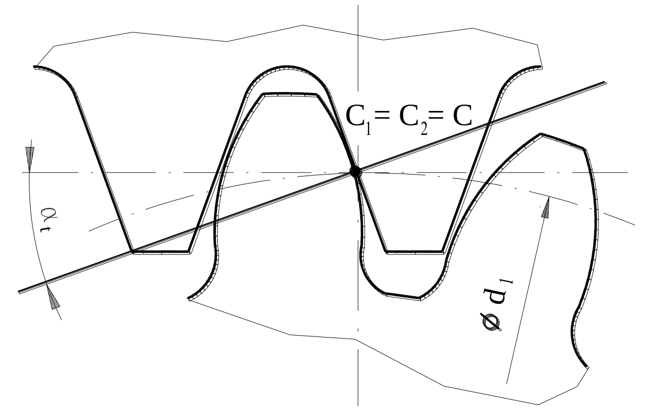


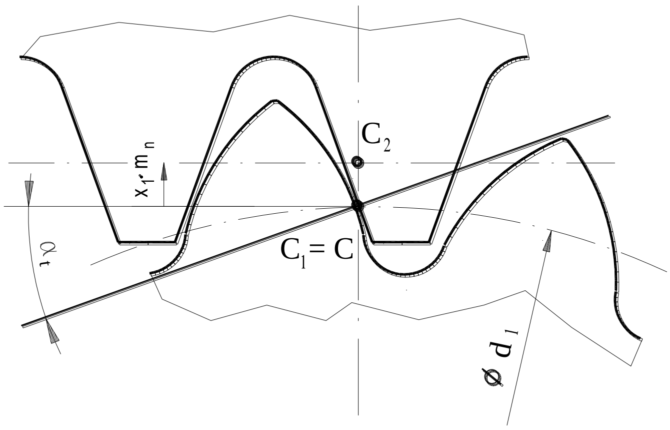
Fig. 2

MESH PINION – RACK

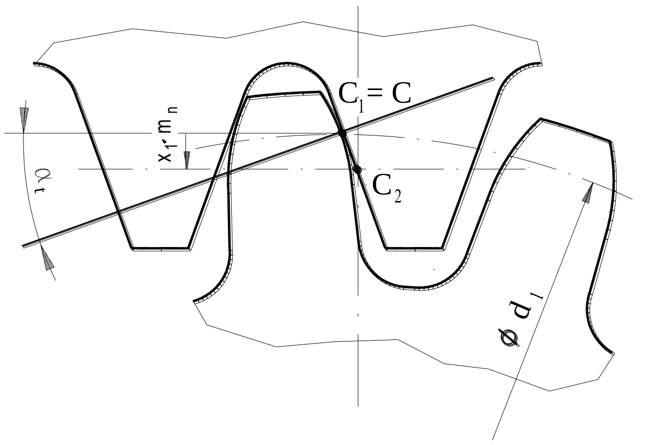
It is necessary to realize that a rack cannot be shifted (it is impossible to use other part of an involute with a different curvature). Due to it is always valid $x_2 = 0$. The next constant is working pressure angle which is independent on pinion addendum coefficient modification x_1. It applies $\alpha_{wt} = \alpha_t$. All detail are on the fig. 3.



$x_1 = 0$
$O_1C = 0{,}5 \cdot d_1$
$O_1C_2 = 0{,}5 \cdot d_1$



$x_1 > 0$
$O_1C = 0{,}5 \cdot d_1$
$O_1C_2 = 0{,}5 \cdot d_1 + x_1 \cdot m_n$



$x_1 < 0$
$O_1C = 0{,}5 \cdot d_1$
$O_1C_2 = 0{,}5 \cdot d_1 + x_1 \cdot m_n$

Fig. 3

All important points at the rack tooth flank result from a mesh. They fully correspond with diameters of an ordinary wheel. Instead of diameters (infinitely big ones) heights parameters related to the reference line (point C_2) are used. There are basic dimensions on the fig. 4. Their nomenclature follows. Corresponding wheel nomenclature is in parentheses.

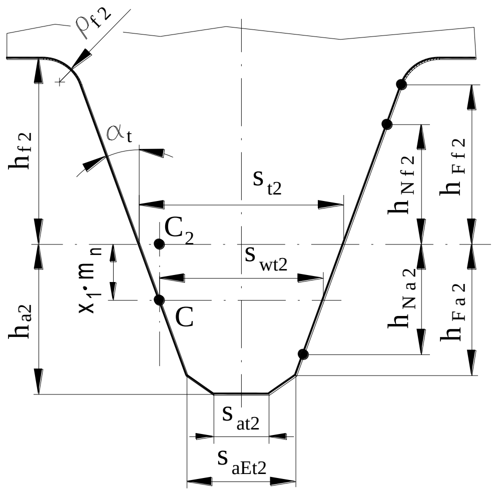
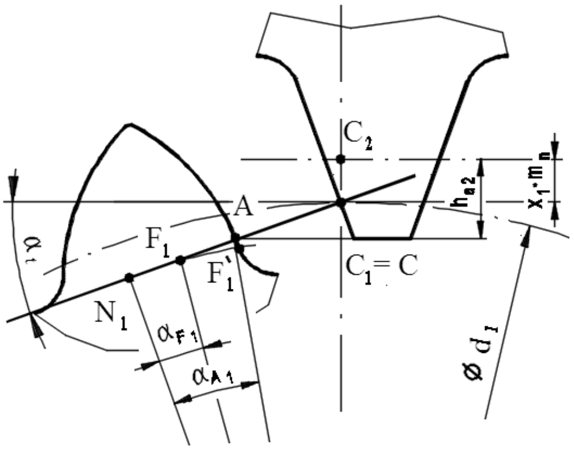


Fig. 4

h_{f2} – root height (root diameter)
h_{Ff2} – root form height (root form diameter)
h_{Nf2} – usable root height (usable root diameter)
h_{Na2} – usable tip height (usable tip diameter)
h_{Fa2} – tip form height (tip form diameter)
h_{aE2} – generated height, usual the same as h_{Fa2} (generated tip diameter)
h_{a2} – tip height (tip diameter)
s_{t2} – transverse reference thickness
s_{wt2} – transverse pitch thickness
s_{aEt2} – transverse thickness at h_{aE2}
s_{at2} – transverse tip thickness

INTERFERENCE

Interference also can occur during pinion – rack mesh. Either on the pinion's root or on rack's root. When a pinion is undercut one (or protuberantly undercut) the interference on pinion's root cannot occur. A rack cannot be undercut by common manufacturing. The undercutting of the rack appears only when it is protuberantly manufactured. And when the rack is protuberantly manufactured – interference cannot occur.



Interference on pinion's root

On the fig.5 relevant diameters goes through points -

F₁ (F'₁) – ϕ d_{F1} (ϕ d_{Ff1})
A₁ – ϕ d_{A1} (ϕ d_{Nf1})
C – ϕ d_1
N₁ – ϕ d_{b1}

Fig. 5

Interference on the pinion's root always occurs when it is valid for angles $\boldsymbol{\alpha_{A1} \leq \alpha_{F1}}$

Diameters d_1 , d_{b1} and d_{F1} which are defined by parameters of the pinion are easy to calculate. For diameter d_{A1} it applies –

$$AC = \frac{h_{a2} - x_1 \cdot m_n}{sin\alpha_t}$$

$$N_1A = 0{,}5 \cdot d_{b1} \cdot \tan \propto_t - AC \quad (1)$$

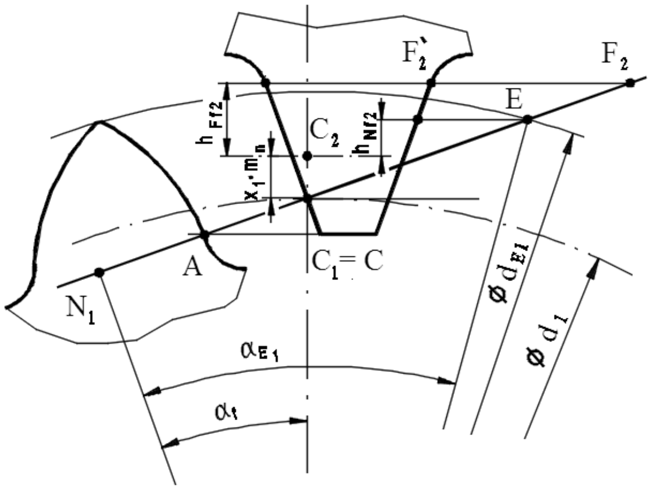
$$d_{A1} = d_{Nf1} = 2 \cdot \sqrt{(0{,}5 \cdot d_{b1})^2 + (N_1A)^2} \quad (2)$$

$$\propto_{A1} = arccos\ \frac{d_{b1}}{d_{A1}} \qquad (3)$$

$$\propto_{F1} = arccos\ \frac{d_{b1}}{d_{F1}} \qquad (4)$$

When chamfering of rack crests is used, instead of addendum height h_{a2} the height to do beginning of chamfering h_{aE2} (h_{Fa2}) is used. On the basis of equations 2,3 and 4 it is easy to enumerate the height h_{Na2} (point A joins with the point F_1 when a pinion is undercut).

Interference on rack's root



Interference on the rack's root always occurs when it is valid **$h_{Ff2} \leq h_{Nf2}$**

Diameters d_1, d_{b1} and d_{E1} which are defined by parameters of the pinion are easy to calculate. For height h_{Nf2} it applies –

$$N_1E = 0{,}5 \cdot d_{b1} \cdot \tan \propto_{E1}$$

$$\propto_{E1} = arccos\ \frac{d_{b1}}{d_{E1}} \qquad (5)$$

Fig. 6

$$CE = N_1E - N_1C = 0{,}5 \cdot d_{b1} \cdot (\tan \propto_{E1} - \tan \propto_t)$$

$$h_{Nf2} = CE \cdot \sin \propto_t - x_1 \cdot m_n \qquad (6)$$

When chamfering of pinion crests is used, instead of tip diameter d_{a1} for finding of diameter d_{E1} the diameter of beginning of chamfering d_{aE1} (d_{Fa1}) is used. It is also easy to enumerate diameter d_{a1max} at the interference limit. When rack is protuberantly undercut the interference cannot occur.

CONTACT RATIO

Transverse contact ratio ε_α is relation between length of a meshing line and transverse pitch on base diameter p_{bt}. Length of a meshing line is defined by points A and E as it is seen on the fig. 6. It applies that this length can be enumerate as $AE = N_1E - N_1A$. Where N_1A is in the equation (1).

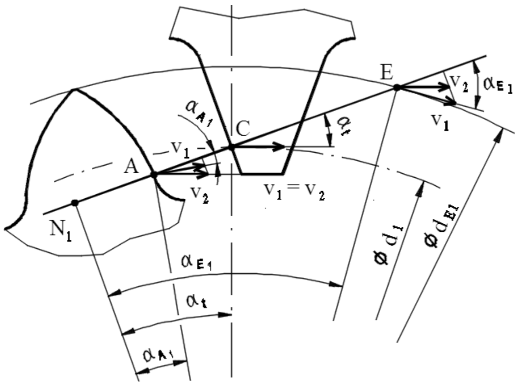
$$N_1E = 0{,}5 \cdot d_{b1} \cdot \tan \propto_{E1} = \sqrt{(0{,}5 \cdot d_{E1})^2 - (0{,}5 \cdot d_{b1})^2}$$

$$\varepsilon_\alpha = \frac{N_1E - N_1A}{p_{bt}} = \frac{N_1E - N_1A}{\pi \cdot m_n \cdot \cos \alpha_t} \cdot \cos \beta \qquad (7)$$

Overlap ratio ε_β is enumerated consistently with ordinary helical gearings.

$$\varepsilon_\beta = \frac{b \cdot \sin \beta}{\pi \cdot m_n} \qquad (8)$$

RELATIVE SLIDINGS



For rack applies that it has the same straight velocity v_2 in each point of the mesh. This figures out from a circumferential speed of the pinion at the reference circle. Both velocities are the same at the meshing point C and they have the same direction too

→ $v_1 = v_2$. See fig.7.

Fig. 7

For a relative sliding at pinion's root (point A) it applies for sliding velocities (perpendicular to the meshing line) –

$$\vartheta_{A1} = \frac{v_{1tA1} - v_{2t}}{v_{1tA1}} = \frac{v_1 \cdot \sin\alpha_{A1} - v_2 \cdot \sin\alpha_t}{v_1 \cdot \sin\alpha_{A1}} = \frac{r_{A1} \cdot \omega_1 \cdot \sin\alpha_{A1} - r_1 \cdot \omega_1 \cdot \sin\alpha_t}{r_{A1} \cdot \omega_1 \cdot \sin\alpha_{A1}}$$

After modifying –

$$\vartheta_{A1} = 1 - \frac{d_1}{d_{A1}} \cdot \frac{\sin\alpha_t}{\sin\alpha_{A1}}$$

Pinion's tip (point E) –

$$\vartheta_{E1} = 1 - \frac{d_1}{d_{E1}} \cdot \frac{\sin\alpha_t}{\sin\alpha_{E1}}$$

Wheel's root (point E) –

$$\vartheta_{E2} = 1 - \frac{d_{E1}}{d_1} \cdot \frac{\sin\alpha_{E1}}{\sin\alpha_t}$$

Wheel's tip (point A) –

$$\vartheta_{A2} = 1 - \frac{d_{A1}}{d_1} \cdot \frac{\sin\alpha_{A1}}{\sin\alpha_t}$$

Addendum modification coefficient for balanced specific slidings

To get the same relative slidings at pinion's and rack's roots (or tips) can apply by suitable choice of pinion's AMD (AMD for a rack is always zero). If relative slidings at roots are the same, relative slidings at tips are the same by themselves. Deriving is made for roots.

$$\vartheta_{A1} = \vartheta_{E2} \quad \rightarrow \quad 1 - \frac{d_1}{d_{A1}} \cdot \frac{\sin\alpha_t}{\sin\alpha_{A1}} = 1 - \frac{d_{E1}}{d_1} \cdot \frac{\sin\alpha_{E1}}{\sin\alpha_t}$$

$$(d_1 \cdot \sin\alpha_t)^2 - (d_{E1} \cdot \sin\alpha_{E1}) \cdot (d_{A1} \cdot \sin\alpha_{A1}) = 0$$

As far as gearing without undercutting, protuberance and chamfering think of, it is possible to use $d_{E1} = d_{a1}$. The diameter d_{A1} is derived from the rack's height h_{a2}, see fig 5 and equation (2).

$$d_{E1} = d_{a1} = d_1 + 2 \cdot (h^*_{a1} + x_1) \cdot m_n$$

$$d_{A1} = 2 \cdot \sqrt{\left(\frac{d_{b1}}{2}\right)^2 + \left(\frac{d_{b1}}{2} \cdot \tan\propto_t - \frac{h_{a2} - x_1 \cdot m_n}{sin\alpha_t}\right)^2}$$

After substitution –

$$(d_1 \cdot \sin\alpha_t)^2 - \left((d_1 + 2\cdot(h^*_{a1}+x_1)\cdot m_n)\cdot sin\left(arccos\,\frac{d_{b1}}{d_1+2\cdot(h^*_{a1}+x_1)\cdot m_n}\right)\right)\cdot$$

$$\cdot\left(2\cdot\sqrt{\left(\frac{d_{b1}}{2}\right)^2+\left(\frac{d_{b1}}{2}\cdot\tan\propto_t-\frac{h_{a2}-x_1\cdot m_n}{sin\alpha_t}\right)^2}\right. \qquad (9)$$

$$\left.\cdot\, sin\left(arccos\,\frac{d_{b1}}{2\cdot\sqrt{\left(\frac{d_{b1}}{2}\right)^2+\left(\frac{d_{b1}}{2}\cdot\tan\propto_t-\frac{h_{a2}-x_1\cdot m_n}{sin\alpha_t}\right)^2}}\right)\right)=0$$

Although the resulting homogeneous nonlinear equation is extensive, its numerical solution is not difficult. For the sample solution is designed for a couple with a standard profile and with straight teeth, number of teeth of the pinion $z_1 = 20$.

For a pinion with no AMC the value of specific sliding at a root is extremely high –

$f(x_1)$

$x_1 = 0 \quad \rightarrow \quad \upsilon_{A1} = -5{,}890$

After calculation for balanced specific slidings (see fig. 8) –

$x_1 = 0{,}4429 \quad \rightarrow \quad \upsilon_{A1} = -0{,}909$

x_1

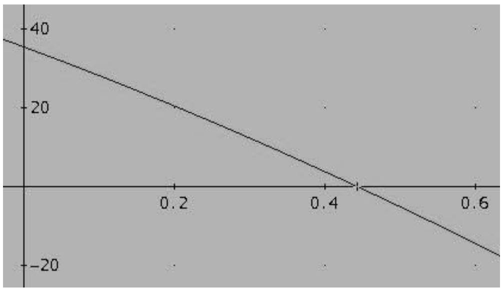


Fig. 8

CONCLUSIONS

Regard to the relatively widespread use of the racks in engineering it is helpful to use for the calculation of basic geometric and meshing parameters exact formulae. Substitution a rack along with a wheel with large number of teeth in the calculations is for many cases insufficient. In precision engineering (machine tools, measuring instr.), it is desirable to use precise calculations. This article presents them.

REFERENCES

1. Litvin, F., L., Fuentes, A.: *"Gear Geometry and Applied Theory."* Second Edition. CAMBRIDGE UNIVERSITY PRESS. ISBN 0-521-81517-7
2. Němček, M.: Návrhový výpočet pro ozubený hřeben*. Sborník mezinárodní XLIX. konference kateder částí a mechanismů strojů.* Str. 215-219. Srní 8. 9. – 10. 9. 2008. ISBN 978-80-7043-718-6.
3. Šalamoun, Č., Suchý, M.: "*Čelní a šroubová soukolí s evolventním ozubením.*" SNTL Praha 1990. ISBN 80-03-00532-9.

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