IMPACT OF DESIGN CONSTRAINTS ON THE SPUR GEAR PAIR PARAMETERS

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Abstract
During the gearbox design process, pinion and wheel parameters are selected so the transmission meets the client needs. The process is rather simple and often automated. However, when the special demands like the limited dimensions or weight are present, task becomes more complex, requiring systematic approach. In this paper we observe the impact of limited outer dimensions on the spur gear pair parameters. To find the optimal solution, we used the genetic algorithm optimization method and ISO standard for load capacity calculation. The gear module, the pinion number of teeth and the both profile shift coefficients were used as variables. The influence of constrained dimensions was examined on two different sets of input data; each consisting of outer gearbox dimensions, rotational speed and desired gear material, representing pairs for different uses.

Key words: *gear; design; constraints; genetic algorithm; optimization.*

INTRODUCTION

The gearbox design process is well known, mostly due to the numerous technical literature. Specialized manufacturers frequently use the automated software to adjust product design to the client needs, and are able to provide 3D models almost instantly. Problem arises when the special design constraints exist. If gearbox dimensions are limited or a maximum permissible weight is set, design changes accordingly and standard approaches are not applicable. Providing the best technical characteristics while meeting the additional constraints is a complex task, which requires a systematic approach. Even though experienced engineer can solve said task by using iterative methods and utilizing his experience, it is often a time consuming process. Since increased time-to-built frequently results in an inferior gearbox unit, optimized design process is required *(Almasi, 2014)*.

To shorten the design time, gear optimization problem is often solved using the genetic algorithm. Genetic algorithm (GA) is an optimization method inspired by the evolution theory, suitable for solving complex technical problems. Since proven to be applicable to the gear optimization problems *(Marcelin, 2001, 2005; Yokota, Taguchi, & Gen, 1998)*, it was regularly used for solving tasks ranging from the gear train weight optimization and preliminary gearbox design *(Gologlu & Zeyveli, 2009; Tudose, Buiga, Ştefanache, & Sóbester, 2010)* to altering the gear micro geometry *(Bonori, Barbieri, & Pellicanos, 2008)*.

In this article, we examined the impact of design constraints on the optimal spur gear parameters. Influence of the constraints restricted gearbox housing dimensions was investigated. After the initial considerations, the authors developed the approach which determines the optimal spur gear pair parameter values by means of GA. Loading capacity was calculated according to ISO 6336:2006 standard and both the tooth root strength and surface durability were calculated. Outer gearbox housing dimensions were used as input. The aim of this study is to enable the better space planning and management to product designers not versed in transmission design.

MATERIALS AND METHODS

Optimization process consists of the GA and ISO 6336:2006 standard for load capacity calculation. The set of constraints includes the user-provided housing dimensions. Gearbox housing shape is simplified; assumed to be a hollow square prism. Design guidelines (1) limit the housing wall thickness and gear tip to housing distance and are used to calculate the available volume to accommodate the gear pair (2). GA was then used to find the pair parameters enabling the highest operational torque, while meeting the

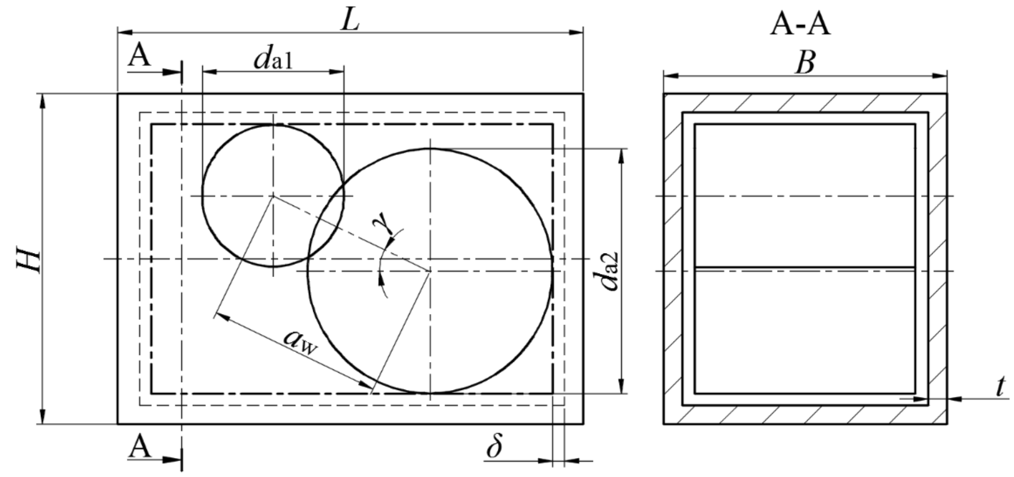

Fig. 1 Simplified gearbox

necessary dimension criteria. Input parameters comprise of the gear pair and shaft material properties, gear quality grade, rotational speed, transmission ratio, application factor and the largest acceptable housing dimensions. To avoid limiting the optimization space, centre distance a_w is taken as a non-standard value. Two shaft arrangements (Fig. 1) can be observed – with axes located in the horizontal ($\gamma = 0°$) or angled plane ($\gamma \neq 0°$). Largest plane angle value γ is limited by the available length l (2) and height h (2); occurring when the pinion addendum diameter tangents the upper or lower boundary plane, while the wheel tangents the opposite one (3). For a unit to be well-designed, gear pair has to satisfy the length condition (4). Pair width is equal to the available width b, but not larger then 25m to ensure the proper face load distribution.

$$\delta = t = 2 + 0.025 \cdot a_w; \qquad t, \delta \geq 8 \tag{1}$$

$$l \times h \times b = \begin{cases} b = B - 2(\delta + t) \\ l = L - 2(\delta + t) \\ h = H - 2(\delta + t) \end{cases} \tag{2}$$

$$\gamma_{max} = \arcsin\left(\frac{h}{a_w} - \frac{d_{a1} + d_{a2}}{2a_w}\right). \tag{3}$$

$$\frac{d_{a1} + d_{a2}}{2} + a_w \cos\gamma_{max} \leq l \tag{4}$$

Shafts are considered to be smooth and made of gear material to enable more compact design. Permissible shaft stress is found using to the expression $\sigma_P = \sigma_f /4$ by *(Haberhauer & Bodenstein, 2014).* To ensure viability of the solutions, necessary shaft diameters are calculated and compared against the pinion dedendum diameter (5):

$$d_{shaft1} = \sqrt[3]{\frac{32M_{red}}{\pi\sigma_P}} \leq m \cdot [z_1 + 2 \cdot x_1 - 2 \cdot (1 + c)] \tag{5}$$

The optimization vector comprises of the four variables: gear module m, number of teeth (pinion) z_1 and profile shift coefficient of pinion x_1 and wheel x_2, with gear module values chosen from the standard. Furthermore, boundary conditions are set for each of the variables to ensure the industrially applicability of solutions. Initial population size was 300 and 1000 generations are calculated. Mutation rate of 0.55 is used with two elite chromosomes. Lastly, the objective for both types of constraints is to increase the operational torque T, which serves as the fitness function.

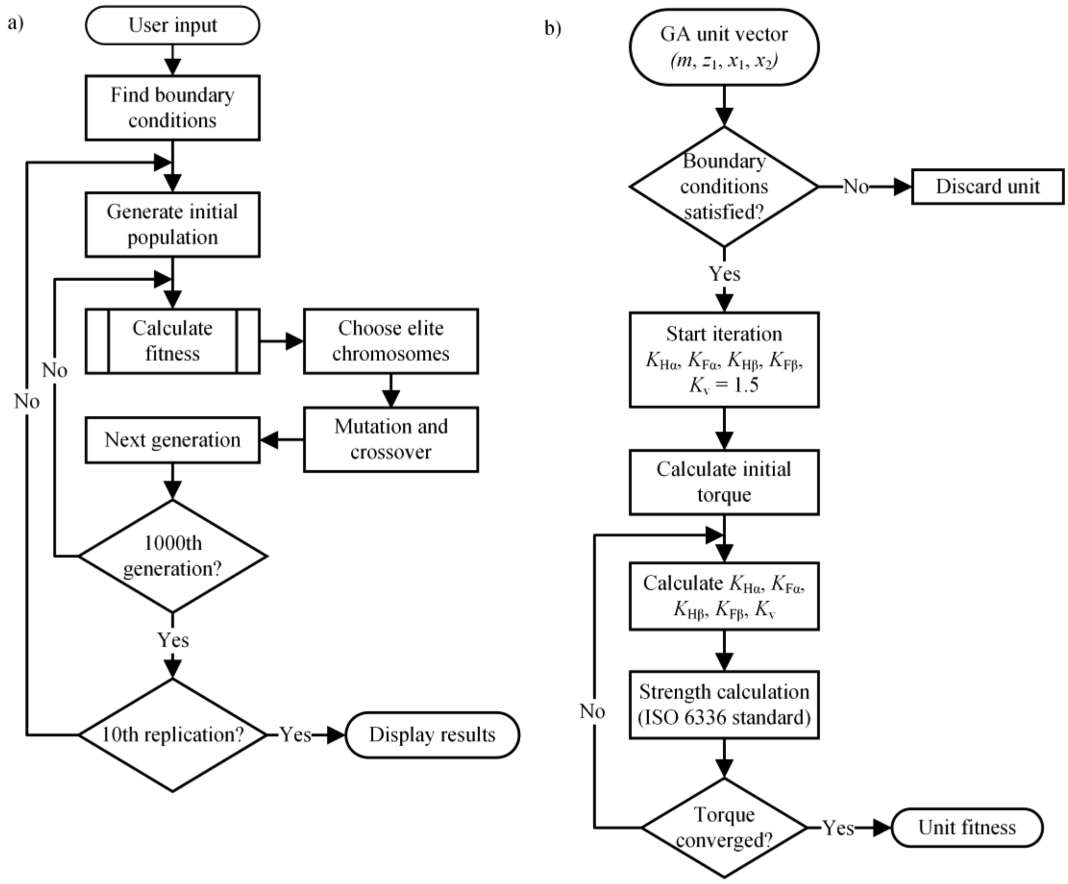

Fig. 1 Optimization process (a) and unit fitness calculation (b)

or calculated torques, the equations (6) and (7) derived from the ISO 6336:2006 standard ensure the necessary load capacity of both the tooth surface and root. Process algorithm is shown in the Fig. 2a and calculation of the unit fitness in the Fig. 2b. The approach was tested on two datasets (Tab. 2) made of 18CrNiMo7-6 and 34 CrMo 4 steel respectively. Permissible operational torque is calculated according to (6) and (7). Permissible contact pressure is responsible for the T_{maxH} and tooth root stress for the T_{maxF}. Lower one of the T_{maxH} and T_{maxF} is chosen as a unit fitness:

$$T_{\mathrm{maxH}} = \frac{m^2 z_1 z_2 b \sigma_{\mathrm{HP}}^2}{2 Z_{\mathrm{B}}^2 Z_{\mathrm{E}}^2 Z_{\mathrm{H}}^2 Z_{\varepsilon}^2 K_{\mathrm{A}} K_{\mathrm{H}\alpha} K_{\mathrm{H}\beta} K_{\mathrm{V}} (i+1)} \tag{6}$$

$$T_{\mathrm{maxF}} = \frac{m^2 z_1 b \sigma_{\mathrm{FP}}}{2 Y_{\mathrm{F}} Y_{\mathrm{S}} Y_{\mathrm{B}} Y_{\mathrm{DT}} K_{\mathrm{A}} K_{\mathrm{F}\alpha} K_{\mathrm{F}\beta} K_{\mathrm{V}}} \tag{7}$$

Tab. 1 Example input data and boundary conditions

		Set 1 (18CrNiMo7-6)	Set 2 (34 CrMo 4)
Rotational speed (input)	n_1, min^{-1}	970	1470
Transmission ratio	i	2.8	3.55
Allowable stress number (bending)	σ_{Flim}, N/mm^2	430	290
Allowable stress number (contact)	σ_{Hlim}, N/mm^2	1500	700
IT quality grade		7	8
Application factor	K_{A}	1.1	1.4
Boundary conditions			
Available housing dimensions, mm	$L \times H \times B$	300×230×140	500×360×210

Since the dynamic calculation factors $K_{H\alpha}$, $K_{F\alpha}$, $K_{H\beta}$, $K_{F\beta}$ and K_V are affected by the change of operating torque, calculation is iterative (Fig. 2b). To find the operating torque, initial values of the dynamic factors are assumed to be 1.5. Resulting torque is then used to calculate dynamic factor values. After the 5 iterations, results converged within the acceptable range. Tooth root stress factors Y_F, Y_S, Y_B, Y_{DT} and surface durability factors Z_E, Z_H, Z_ε, are unaffected by the operating torque variations. To account for influence of the single tooth contact, larger one of the factors Z_B and Z_D is included in the calculation. While determining the allowable stresses σ_{HP} and σ_{FP}, factors Y_{NT}, $Y_{\delta relT}$, Y_{RrelT}, Y_X, Z_{NT}, Z_L, Z_V, Z_R, Z_W and Z_X are assumed to be 1 and stress correction factor Y_{ST} is 2. Safety factor values are S_{Hlim} = 1.2 for surface durability and S_{Flim} = 1.5 for tooth root stress. Accuracy grade IT7 is chosen for set 1 and IT8 for set 2.
In order to rate the solutions provided by the genetic algorithm optimization, the use of provided dimensions is evaluated. Since the housing is assumed to be a hollow prism, it is not possible to achieve the full space utilization. Usage of all the three dimensions is assessed by comparing the resulting spur gear pair width, height and length with their available counterparts. Pair dimensions are measured as projections on the corresponding planes. Optimal set of parameter ratio value is 1, meaning that dimension is completely utilized. Furthermore, after determining the optimal parameters, algorithm can be used to suggest the possible reductions of input dimension. Unused length, width or height are found as a differences of available and calculated dimensions.

RESULTS AND DISCUSSION
Suggested approach was used to find the results. Optimization process was replicated 10 times for each of the datasets, with different initial populations to ensure that results do not represent the local, but a global maximum. 1000 generations were calculated for both sets and solutions converged after the 200 generations, while the changes between the 200th and 1000th generation were minor. Even though set 1 (Fig. 3a) displayed faster rate of convergence between the 1st and 70th generation, after the 100 generations, solutions were satisfying. Rate of convergence can be further increased by using Sobol quasi-random distribution of initial population *(Maaranen, Miettinen, & Penttinen, 2007)*. Mean operational torque value across the 10 replications was calculated and its convergence from generation to generation is shown in the Fig. 3. Logarithmic scale was used in order to better display the changes during the initial generations

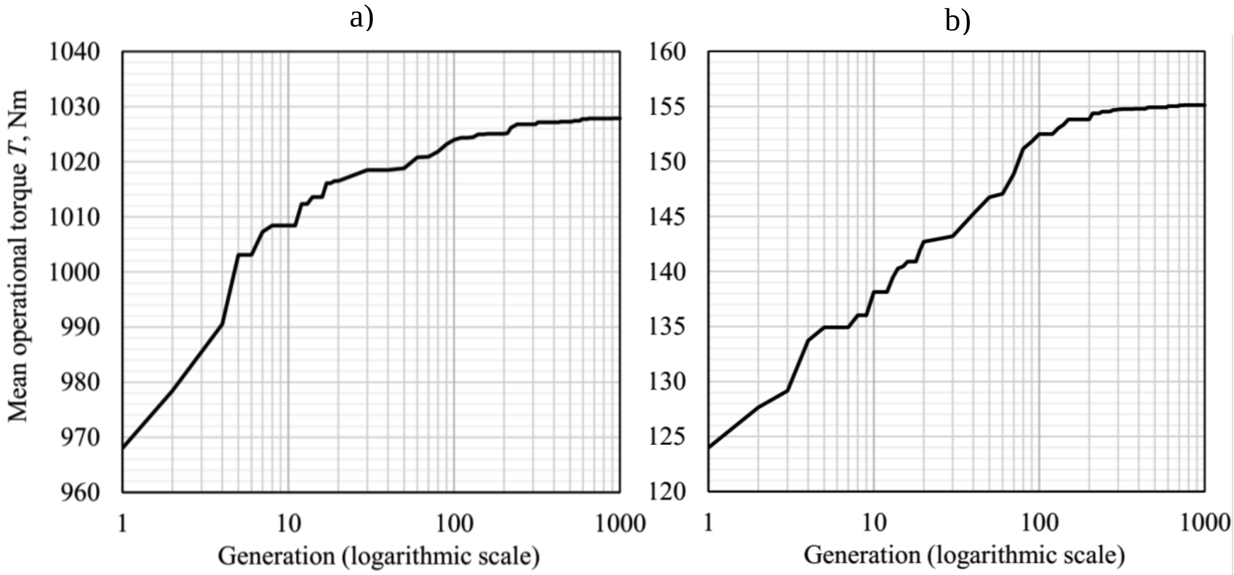

Fig. 3 Convergence diagrams for the both datasets – set 1 (a) and set 2 (b)

Tab. 2 Resulting spur gear pairs

	Set 1	Set 2
Transmissible operational torque, Nm	1029	155.6
Number of teeth (pinion)	18	15
Number of teeth (wheel)	50	53
Gear module, mm	3.75	6
Profile shift coefficient (pinion)	0.577	0.490
Profile shift coefficient (wheel)	-0.065	-0.216
Face width, mm	97.5	138
Pinion dedendum diameter, mm	62.45	80.88
Approximate pinion shaft diameter, mm	57.28	36.70

The spur gear pairs with the highest fitness are shown in the Tab 2. All the replications converged towards the same gear module and number of teeth. Differences were encountered among the profile shift coefficients, which were set as continuous instead of discrete variables. Among the replications, largest operational torque difference was 0.334% for data set 1, and 0.962% for set 2, with standard deviations of 1.189 Nm and 0.292 Nm respectively. Result dispersion could be further lowered by discretization of the profile shift coefficients.
Algorithm choses the gear module and pinion number of teeth that fit into the user-provided dimensions. To fill the rest of the available space, profile shift is used. Even though the larger wheel profile shift will result in the increased operational torque, it will also cause the wheel addendum diameter to exceed the available dimensions. Pinion profile shift coefficient was chosen as a largest allowable value considering the required tooth tip thickness. It should be noted that even though it is important for the gearing longevity, specific sliding is not considered in this paper.
Result evaluation was carried out by comparing the gear pair length, width and height against their available counterparts (Tab. 3). Height is utilized first and width second, so for the both pairs height ratio is equal to 1. However, pair length and width reductions are possible for the both sets. When analysing the set 2 results, unused length is mainly caused by the poor input data; wheel addendum circle diameter is equal to the available housing height. Fixed addendum diameter coupled with the constant transmission ratio results in a highly constrained pinion, and consequently, pair dimensions. The housing volume could be further lowered by changing its shape, which is currently a hollow prism. While optimizing the housing shape, finite element method (FEM) can be used for static and dynamic analysis *(Weis, Kučera, Pecháč, & Močilan, 2017)*.
Both datasets do not make use of available width. For set 1 cause is the negative shaft length influence on the bending moment. Beside the dimensional constraints, the shaft bending strength is an important condition. Required shaft diameter for set 1 is 5.17 mm lower than the pinion dedendum diameter, meaning that shaft and gear have to be made as a single part. Even though the gear teeth increase the shaft cross section moment of inertia, overlap of the shaft and tooth root bending stresses should be avoided not to cause local critical stresses. Furthermore, the sum of profile shift coefficients also influences the shaft bending moment; increased sum would result in a larger pressure angle at the pitch cylinder (α_w) responsible for the radial component of force. On the other hand, width of set 2 is limited by the empirical guideline; maximal recommended gear width should not exceed the 25 m. Since m is limited by addendum diameter equal to the available height, further increase in width is not possible even though input dimensions would allow it.

Tab. 3 Result evaluation

	Set 1			Set 2		
	Used	Available	Ratio	Used	Available	Ratio
Pair width, mm	97.5	108	0.903	150	178	0.843
Pair height, mm	198	198	1	328	328	1
Pair length, mm	250.9	268	0.936	391.1	468	0.836

CONCLUSIONS

The spur gear pair design process with special constraints was carried out. Our main goal was to develop the approach which will enable designer to achieve the necessary transmission characteristics in limited space. Proposed approach, which consisted of the genetic algorithm as optimization method and ISO 6336:2006 standard for gear load calculation, was tested using the two arbitrary data sets. Results for both sets were found, and calculation process was replicated 10 times to verify it.

The operational torque was used as the fitness function, which was maximized. Requirement to increase the torque, paired with the static geometrical constraints, directed the algorithm towards fitting the gears pair into the available volume. Limited dimensions coupled with the use of high-quality materials caused the shafts to become critical element in the chain for set 1.

Suggested approach could easily be adjusted to search optimal gear pair parameters for different types of constraints. For example, if the maximal gearbox weight is set as a limiting condition, replacing the dimensional constraints with the ones associated to weight will enable calculation of the corresponding optimal pair parameters.

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